Two-Dimensional ^{3}He: A Crucial System for Understanding Fermion Dynamics

with H. M. Böhm, M. Panholzer (Theory)[1]
and H. Godfrin[2] and H. J. Lauter[3] (Experiment)

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[3]Institut Laue Langevin, F-38042 Grenoble Cedex, France.

MB-15, July 27-31, 2009

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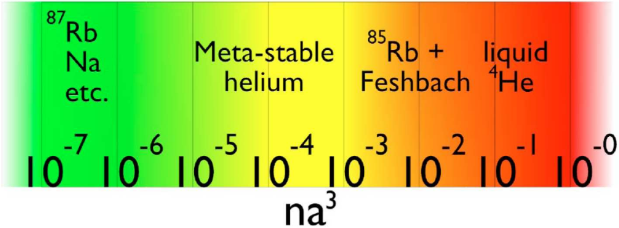
Outline

Jonathan DuBois and Henry Glyde, *Phys. Rev.* **A 68**, 033602 (2003)



... if you can do Helium, many“ cold gas” problems are easy !

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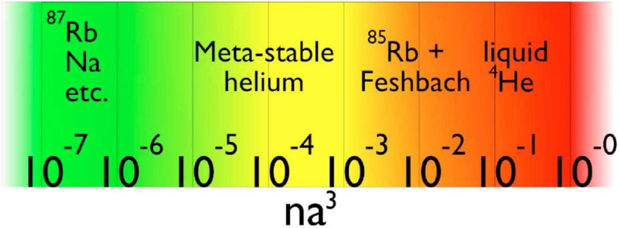

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- ^{4}He is very well understood (See Campbell’s poster)

Why do helium physics ?
When nowadays cold gases are hot stuff

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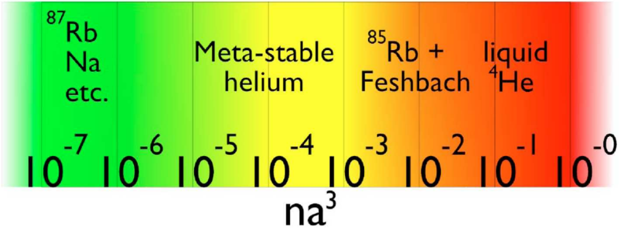
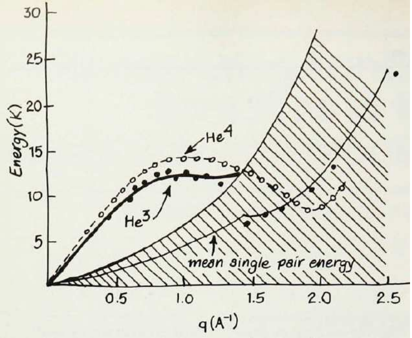


... if you can do Helium, many“ cold gas” problems are easy !

- ^{4}He is very well understood (See Campbell’s poster)
- So let’s look at ^{3}He !

From David Pines, *Physics Today* **34**, 106 (1981):

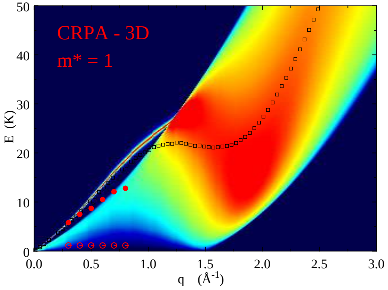


Woods' (A.D.B. Woods, *Phys. Rev. Letters* **14**, 355 (1965).) experiment let me conclude that the phonon-maxon-roton excitation in He II and the zero sound mode of ^{3}He had a common physical origin in strong (and quite similar) effective interatomic interactions . . . .

What we were told in (some) textbooks:

Dynamic structure function:

$$S(q,\omega) = \frac{1}{\pi} \Im m\, \chi(q,\omega)$$



- interested (for the time being) in *density fluctuations;*

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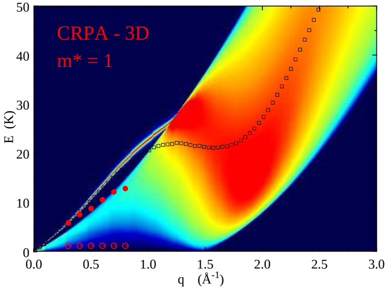

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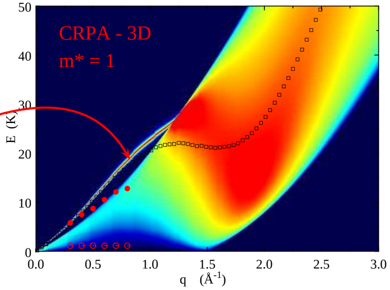
Collective mode at

$$1 - \tilde{V}_{p-h}(q)\chi_0(q,\omega(q)) = 0$$



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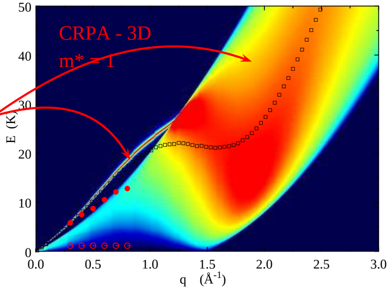
Particle-hole continuum at

$$e(q-k_F) \le \hbar\omega \le e(q+k_F)$$



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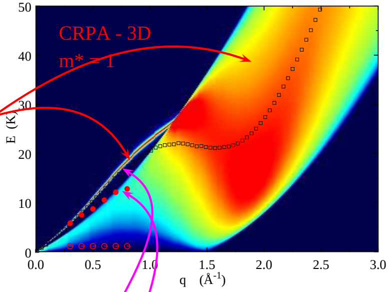
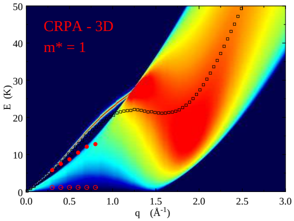


- interested (for the time being) in *density fluctuations;*
- similar effect as for bosons: *RPA is too high compared to experiments.*

Messing with (effective) masses:
the solution (or not ?)

- Recall where we started
  . . . and what goes wrong:



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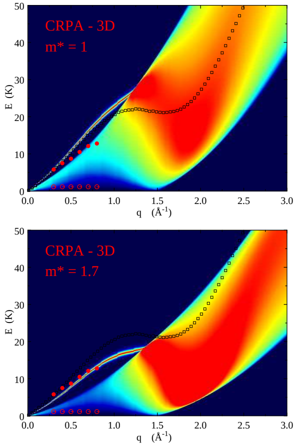
- An effective mass can (potentially) explain $S(q, \omega)$



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- An effective mass can (potentially) explain $S(q,\omega)$
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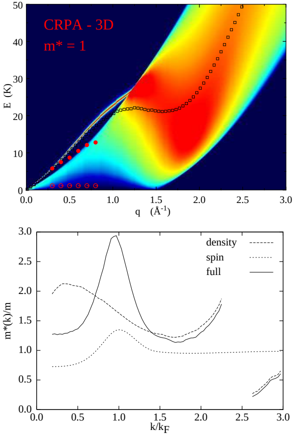

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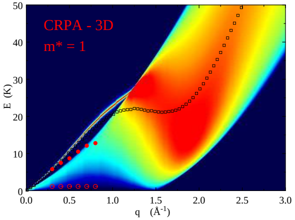
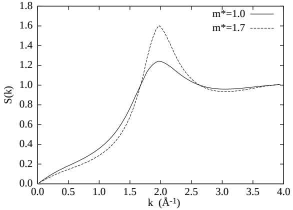
- An effective mass can (potentially) explain $S(q, \omega)$
- BUT the effective mass is far from constant
- BUT an effective mass in RPA messes up sum rules

$$m_0(q) \equiv \int_0^\infty d(\hbar\omega) S(q,\omega) = S(q)$$

$$m_1(q) \equiv \int_0^\infty d(\hbar\omega)(\hbar\omega) S(q,\omega) = \frac{\hbar^2 q^2}{2m}$$





Messing with (effective) masses:
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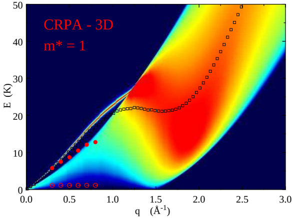
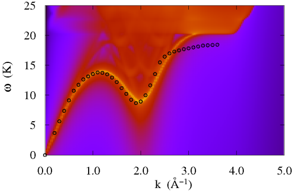
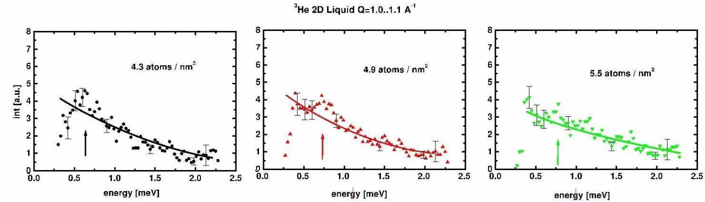
- Recall where we started
  . . . and what goes wrong:
- An effective mass can (potentially) explain $S(q,\omega)$
- BUT the effective mass is far from constant
- BUT an effective mass in RPA messes up sum rules
- BUT we don't need an effective mass in ^{4}He.





$S(k,\omega)$ in two dimensional ^{3}He – the key experiment

ILL/CNRS measurements: Godfrin, Lauter, Meschke



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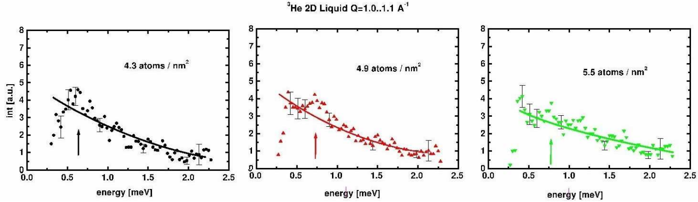
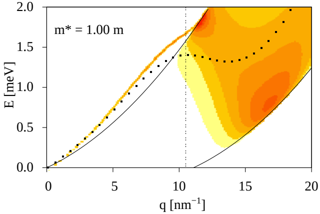
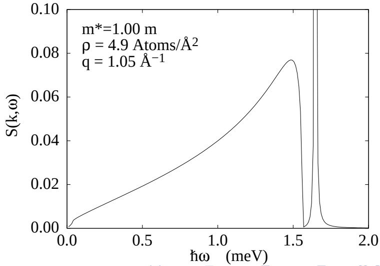


- RPA gives wrong position of the collective mode relative to the continuum;



$$\frac{m^*}{m} = 1$$



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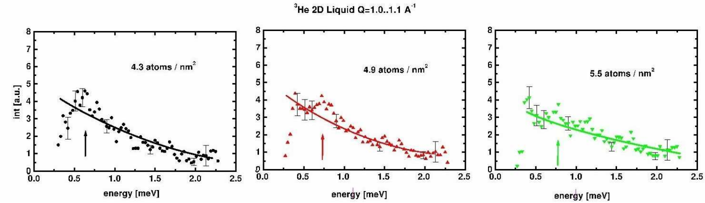
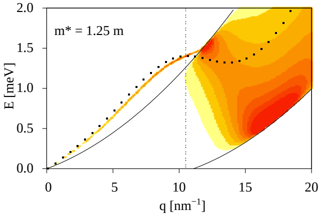
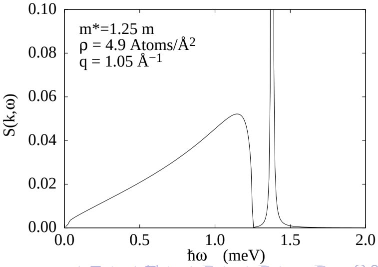


- RPA gives wrong position of the collective mode relative to the continuum;
- Messing with m^* does not help !



$$\frac{m^*}{m} = 1.25$$



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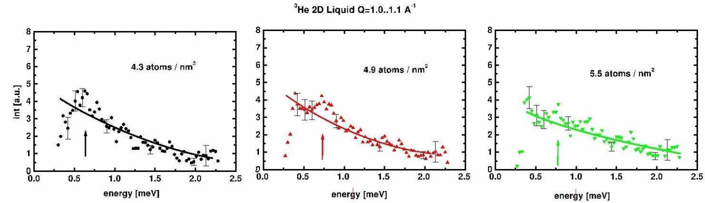

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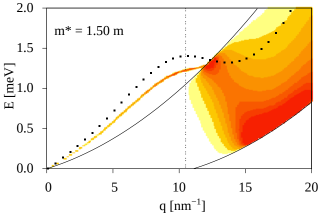
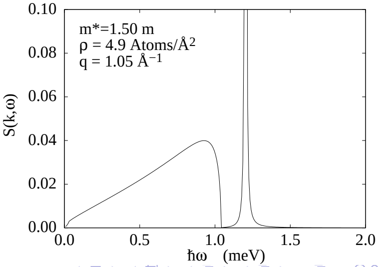


$$\frac{m^*}{m} = 1.50$$



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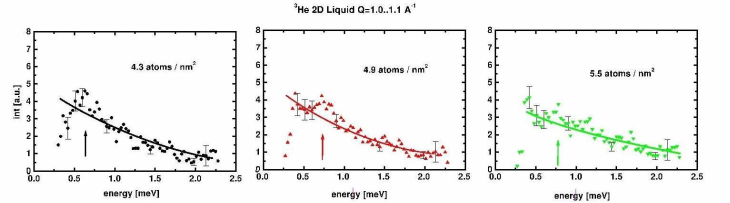

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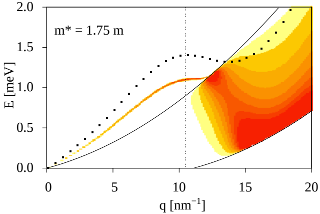
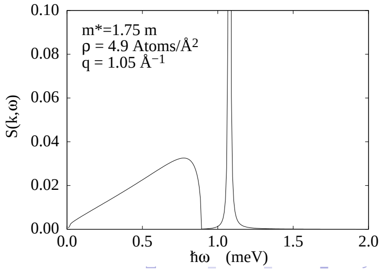


$$\frac{m^*}{m} = 1.75$$



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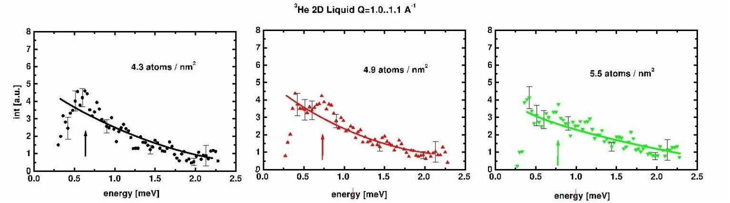

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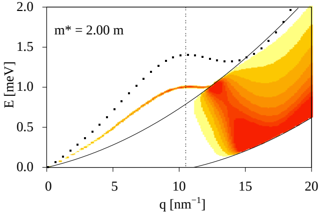
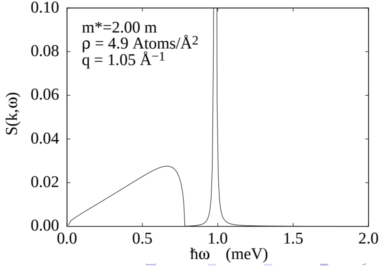


$$\frac{m^*}{m} = 2.00$$



The success story for bosons:
Wave function for excited states:

$$|\Psi(t)\rangle = e^{-iE_0 t/\hbar} \frac{F e^{\frac{1}{2}\delta U} |\Psi_0\rangle}{\langle \Psi_0 | e^{\frac{1}{2}\delta U^\dagger} F^\dagger F e^{\frac{1}{2}\delta U} |\Psi_0\rangle]^{1/2}} ,$$

$|\Psi_0\rangle$: model ground state, $\delta U(t)$: excitation operator
Bosons:

$$\delta U(t) = \sum_i \delta u^{(1)}(\mathbf{r}_i; t) + \sum_{i<j} \delta u^{(2)}(\mathbf{r}_i, \mathbf{r}_j; t) + \ldots$$

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Fermions:

$$\delta U(t) = \sum_{\mathbf{p},\mathbf{h}} \delta u^{(1)}_{\mathbf{p},\mathbf{h}}(t) a^\dagger_{\mathbf{p}} a_{\mathbf{h}} + \sum_{\mathbf{p},\mathbf{h},\mathbf{p}',\mathbf{h}'} \delta u^{(2)}_{\mathbf{p},\mathbf{h},\mathbf{p}',\mathbf{h}'}(t) a^\dagger_{\mathbf{p}} a^\dagger_{\mathbf{p}'} a_{\mathbf{h}} a_{\mathbf{h}'}$$

- The physical content of $\delta u^{(2)}$ is not describable in (dynamic or not) mean field theory !

- $\delta u^{(2)}_{\mathbf{ph},\mathbf{p'h'}}(t) = 0$, $F = 1$, weakly interacting Hamiltonian:

$$\begin{pmatrix} e_{ph} - \hbar\omega + V^{(A)}_{ph',hp'} & V^{(B)}_{pp',hh'} \\ V^{(B)}_{hh,pp'} & e_{ph} + \hbar\omega + V^{(A)}_{hp',ph'} \end{pmatrix} \begin{pmatrix} \delta u^{(1)}_{ph} \\ \delta u^{*(1)}_{ph} \end{pmatrix} = \begin{pmatrix} U^{(ext)}_{ph} \\ U^{*(ext)}_{ph} \end{pmatrix}$$

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- Set $V^{(A)}_{ph',hp'} = V^{(B)}_{pp',hh'} = V(q)$ gives ordinary RPA.

$$\chi(q,\omega) = \frac{\chi_0(q,\omega)}{1 - V(q)\chi_0(q,\omega)}$$

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- Keep $\delta u^{(2)}_{\mathbf{ph},\mathbf{p'h'}}(t)$: Makes all matrix elements energy dependent, does not change the single particle spectrum.

- Assume *local* effective interactions:

$$\begin{pmatrix} e_{ph} - \hbar\omega + V^{(A)}(q,\omega) & V^{(B)}(q,\omega) \\ V^{(B)}(q,-\omega) & e_{ph} + \hbar\omega + V^{(A)}(q,-\omega) \end{pmatrix}$$

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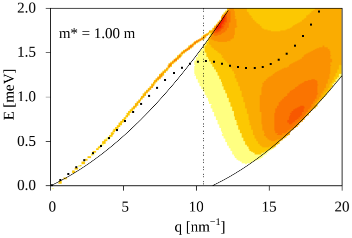
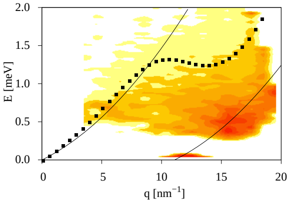
- Keeping exchange gives also self-energy corrections to the spectrum. (Can no longer be built onto Monte Carlo input.)

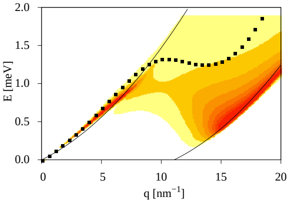
Theory RPA



ILL/CNRS experiment



Theory DMBT



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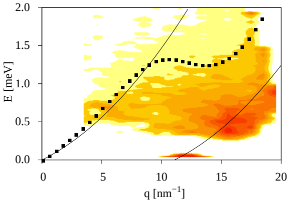
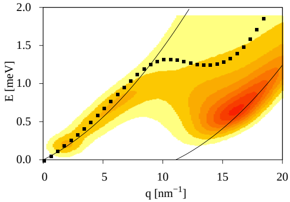


- Pair fluctuations move the zero sound mode to the right energy *without* need to shift the spectrum;

Theory broadened



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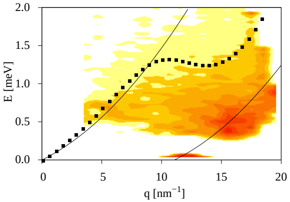
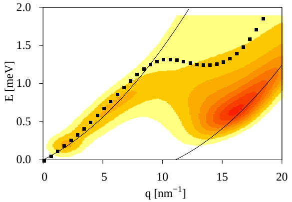
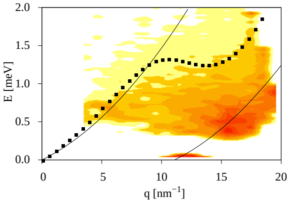


- Pair fluctuations move the zero sound mode to the right energy *without* need to shift the spectrum;
- Including experimental broadening yields excellent agreement;

Theory broadened



ILL/CNRS experiment



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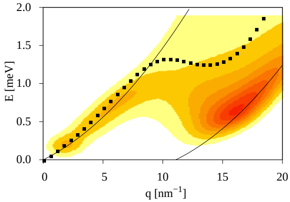
Theory broadened



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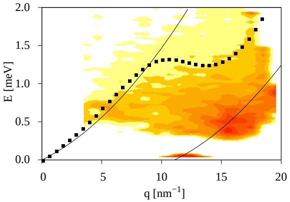
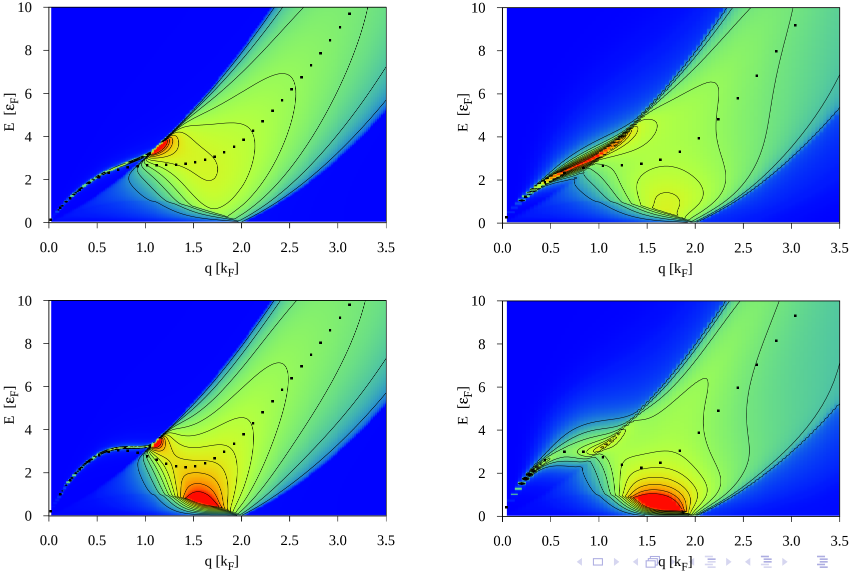


- Pair fluctuations move the zero sound mode to the right energy *without* need to shift the spectrum;
- Including experimental broadening yields excellent agreement;
- We do not claim that proper self-energy inclusions are unimportant;
- Further work is in progress to make the connection between G(0)W and CBF more transparent.

Fermi dipoles

Because everybody talks about cold gases

RPA | CBF



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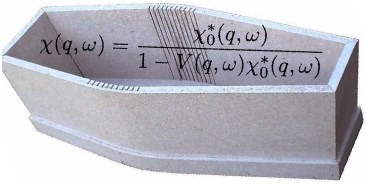

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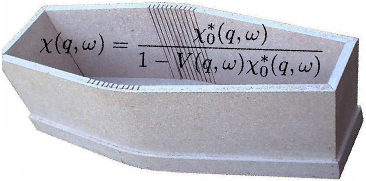

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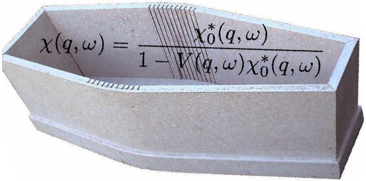

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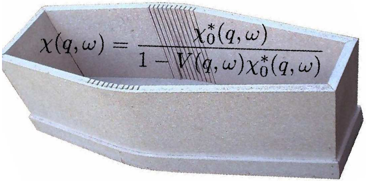

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- Since the new view is experimentally proven for 2D ^{3}He it is natural to review our understanding of other Fermi systems because:
- Getting the right answer does not mean doing the right physics.
- Yes, of course, we can do cold Fermi gases (dipole) gases. . .

Thanks to collaborators in this project

H. M. Böhm	JKU Linz
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