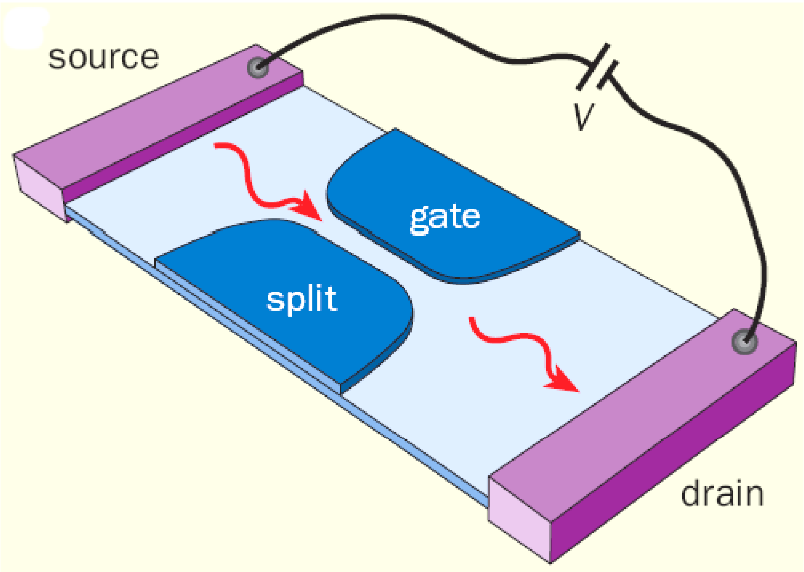
International Conference on Recent Progress in Many-Body Theories
July 27-31, 2009
Ohio State University, Columbus, OH

Critical phenomena in interacting quantum wires



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with
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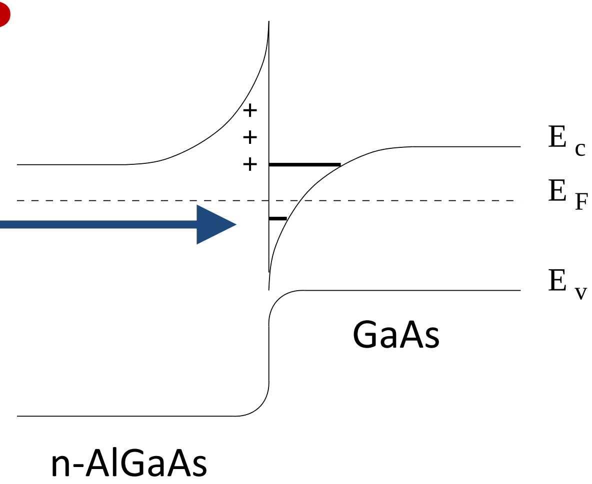
K.A. Matveev (Argonne National Laboratory)

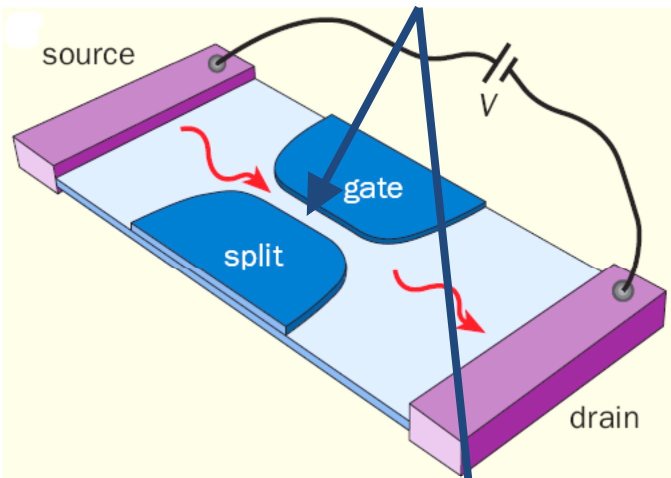
Support: Office of Science, U.S. Department of Energy
DE-FG02-07ER46424

Quantum Wires

- GaAs/AlGaAs heterostructure
  $\rightarrow$ 2D electron gas

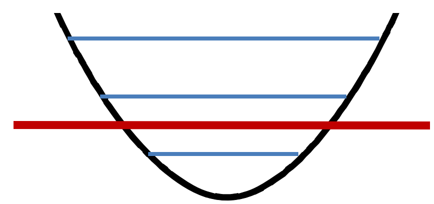


- depletion of the 2D electron gas by gates
  $\rightarrow$ quasi-1D channel



side view:

- confining potential
  in transverse directions
  $\rightarrow$ subband structure

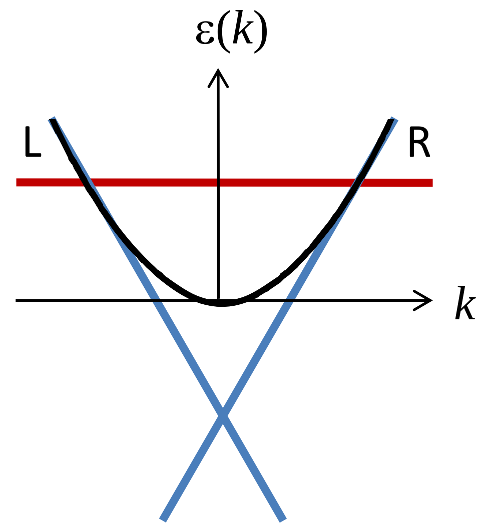


- change chemical potential with gate voltage

Pecularities of one dimension

$d > 1$: Fermi liquid (fermionic quasi-particles)

$d = 1$: Tomonaga-Luttinger liquid
(bosonic collective excitations)



$$H = \frac{\hbar v}{2\pi} \int dx \left(K(\nabla\theta(x))^2 + \frac{1}{K}(\nabla\phi(x))^2 \right)$$

v velocity, K interaction parameter

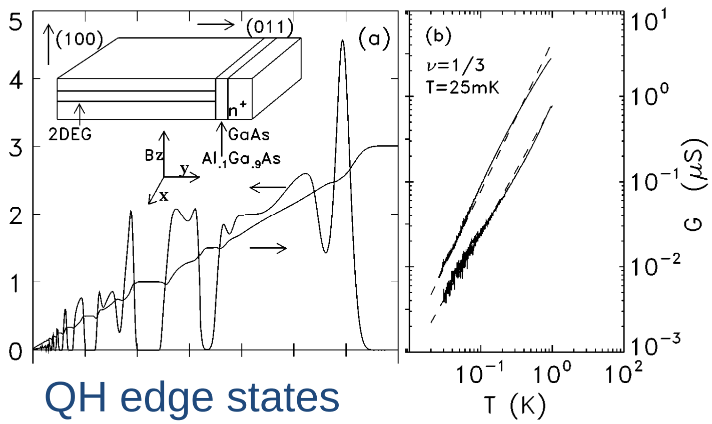
Progress of Theoretical Physics Vol. 5, No. 4, July~August, 1950

Remarks on Bloch's Method of Sound Waves applied to Many-Fermion Problems

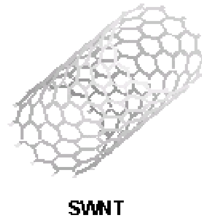
Sin-itiro TOMONAGA

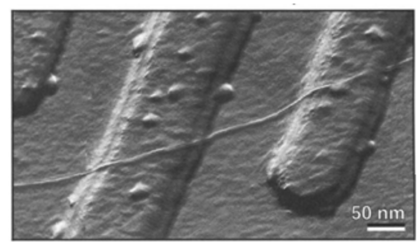
A mathematically closed and clear-cut presentation of the theory is achieved, however, at the expense of physical usefulness, because, thus far, the author has succeded only in giving a complete formulation for a one-dimensional assembly of particles.

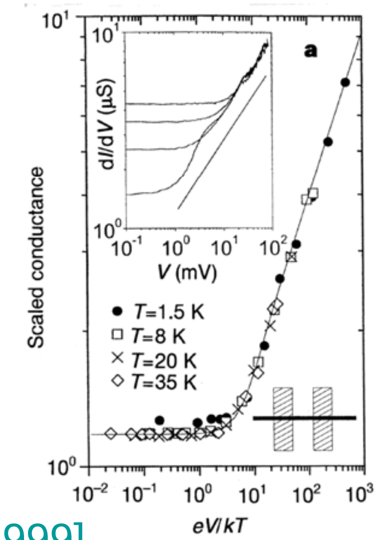
One-dimensional systems



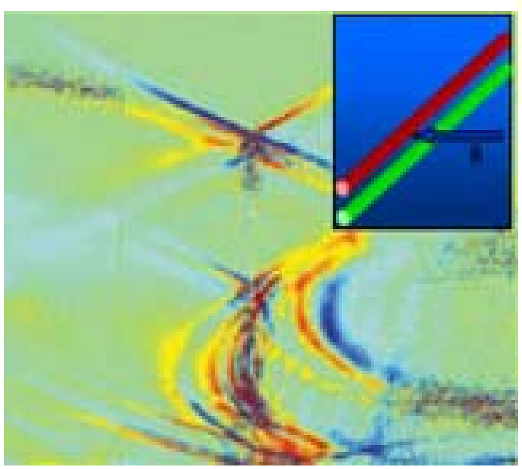
QH edge states
[Chang *et al.* 1996]

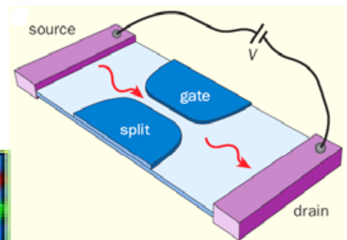


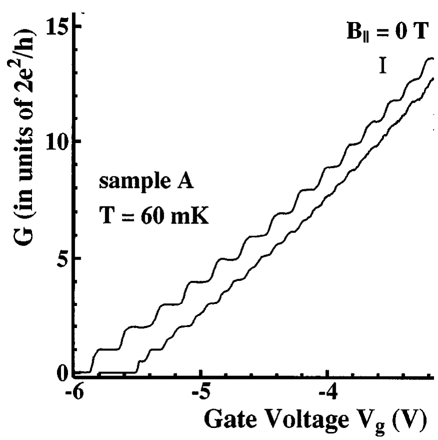




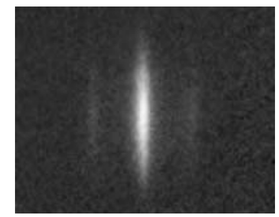
Carbon nanotubes
[Tans *et al.* 1997, Bockrath *et al.* 1999]

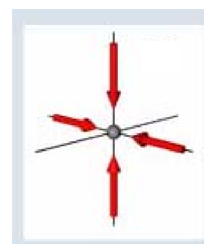


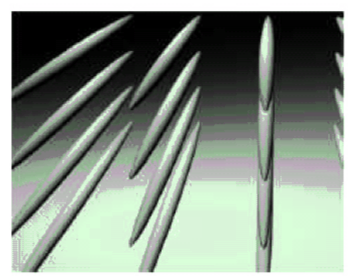


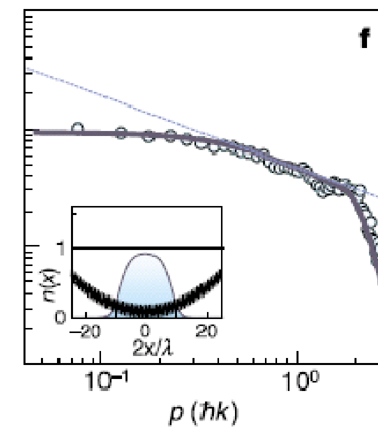


Quantum wires
[Thomas *et al.* 1996, Auslaender *et al.* 2002]







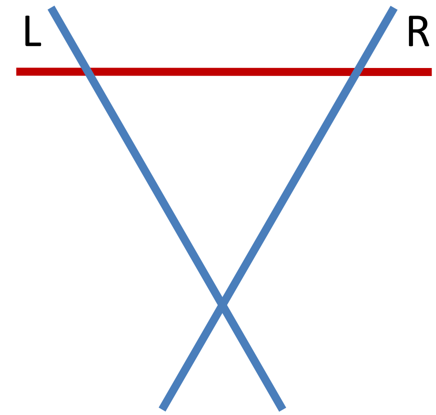


Cold atoms in optical lattices
[Paredes *et al.* 2004]

Pecularities of one dimension

$d > 1$: Fermi liquid (fermionic quasi-particles)

$d = 1$: Tomonaga-Luttinger liquid
(bosonic collective excitations)



$$H = \frac{\hbar v}{2\pi} \int dz \left(K (\nabla\theta(z))^2 + \frac{1}{K} (\nabla\phi(z))^2 \right)$$

v velocity, K interaction parameter

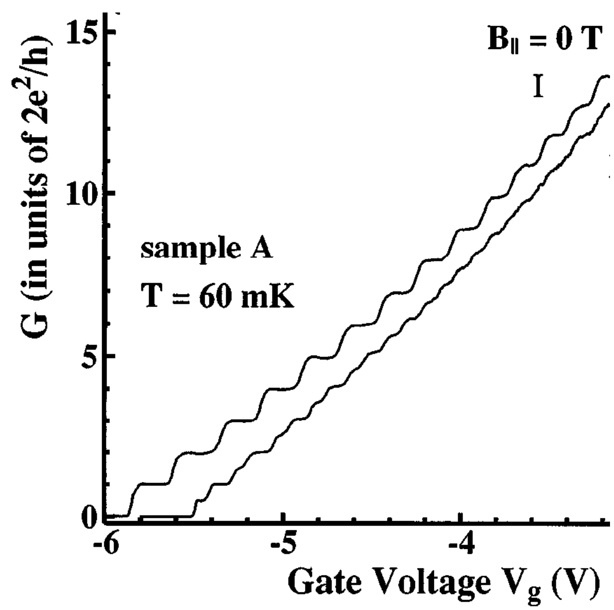
1D beyond Luttinger ?

spin-incoherent Luttinger liquid

Matveev, PRL '04
Cheianov & Zvonarev, PRL '04
Fiete, RMP '07

non-linear Luttinger liquid

Pustilnik et al., PRL '06
Pereira et al., PRL '06
Imambekov & Glazman, Science '09

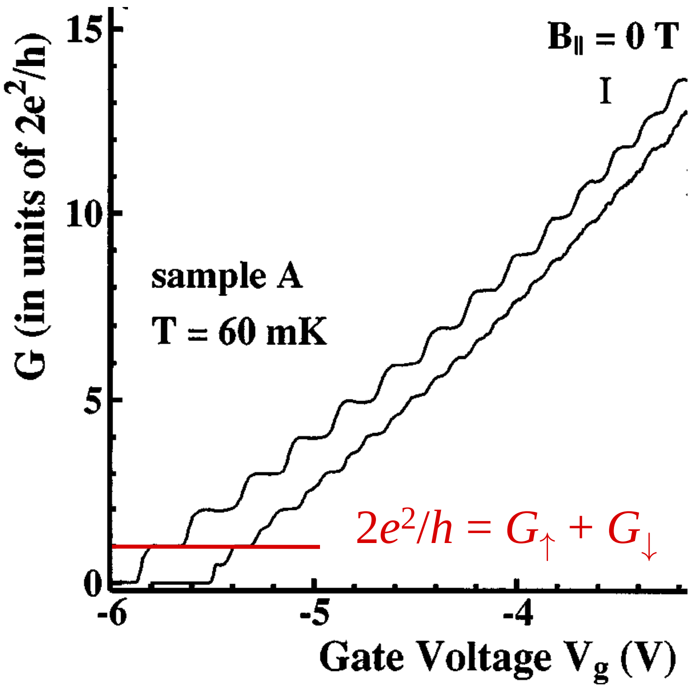


... & beyond strictly 1D ???

The Ohio State University Physics

Conductance quantization in 1D

- $G = k \cdot G_0$ (k integer)
  where $G_0 = e^2/h$

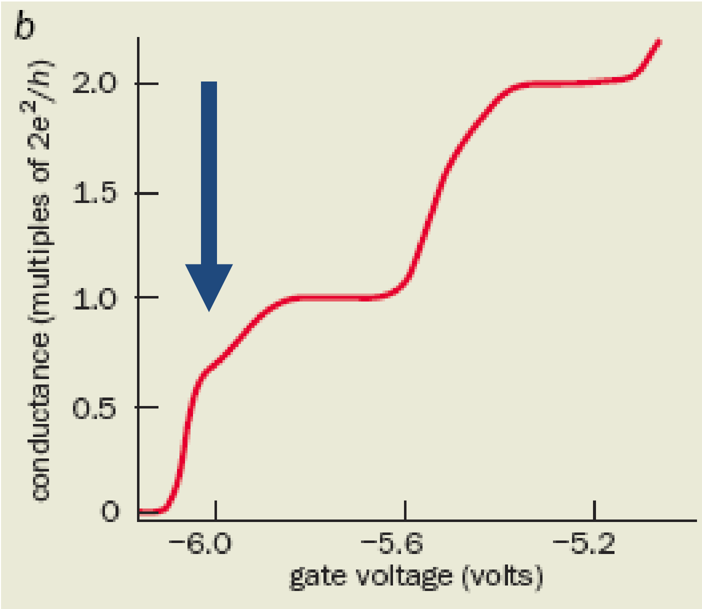


- result not affected by Luttinger liquid interactions
  Maslov & Stone, PRB ’95; Ponomarenko, PRB ’95; Safi & Schulz, PRB ’95

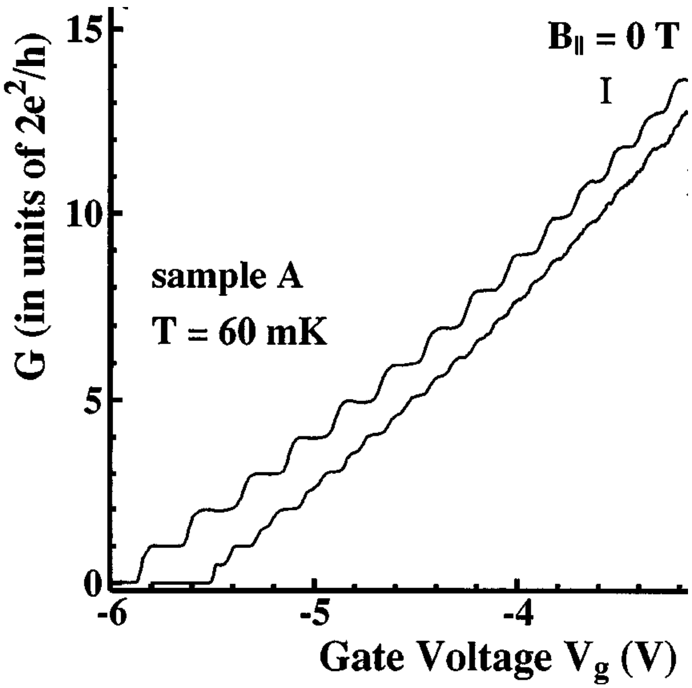
- consecutive filling of **independent** 1D channels

Conductance quantization in 1D

- $G = k \cdot G_0$ (k integer)
  where $G_0 = e^2/h$



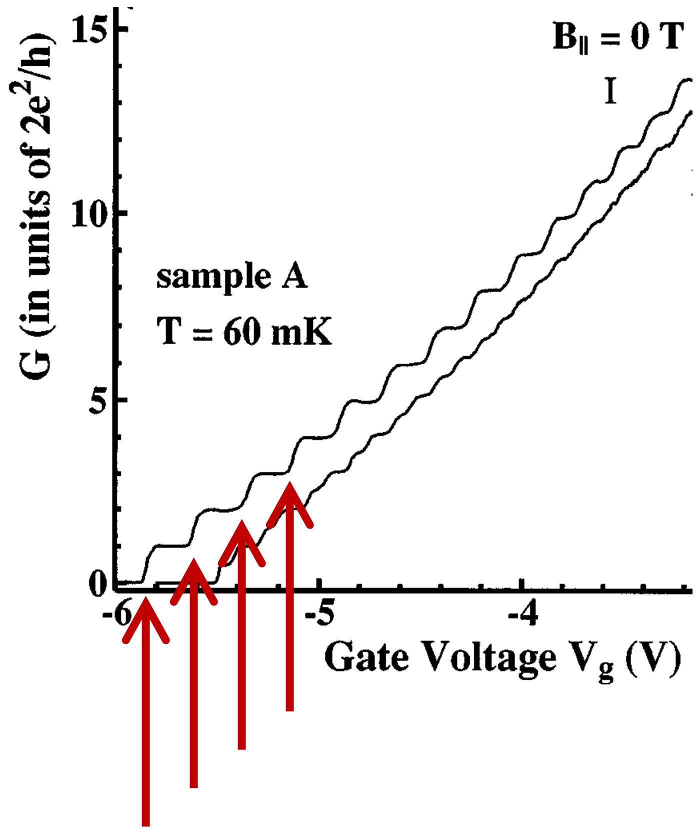
picture taken from
Berggren & Pepper, Physics World 2002



„0.7 structure“

Conductance quantization in 1D

- $G = k \cdot G_0$ (k integer)
  where $G_0 = e^2/h$

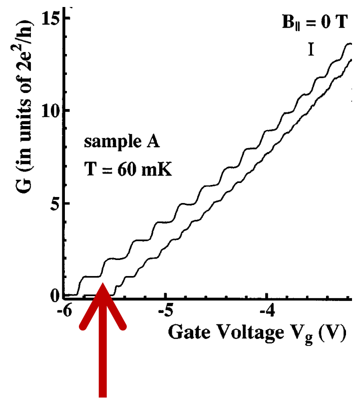


quantum phase transitions

Physics

What is the nature of the phase transition?

What are the properties of the quasi-one-dimensional phase?



Outline

- The model
- Non-interacting electrons   vs   (classical) Wigner crystal
- Phase diagram
- Nature of the transition
- Summary & Outlook

The model

- spinless (= spin-polarized) electrons
- Coulomb interactions
  screened by a gate far from the wire
- parabolic confining potential

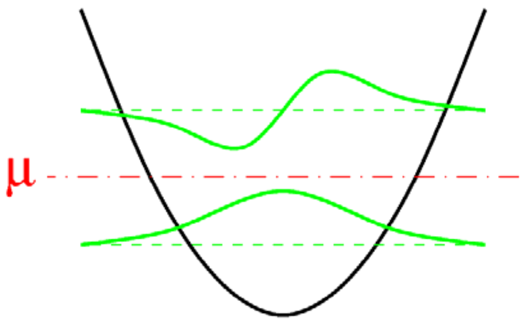
$$
\begin{aligned}
H_{\text{kin}} &= -\int d^2r\ \psi^\dagger \frac{\hbar^2\nabla^2}{2m}\psi \\
H_{\text{Coul}} &= \frac{1}{2}\int d^2r\, d^2r'\ \psi^\dagger(\mathbf{r})\psi(\mathbf{r})V(|\mathbf{r}-\mathbf{r}'|)\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}') \\
H_{\text{conf}} &= \frac{1}{2}m\Omega^2\int d^2r\ y^2\psi^\dagger\psi
\end{aligned}
$$

The model: Subbands

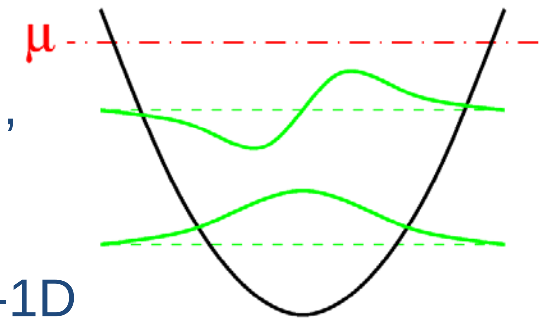
- "not too strong" interactions:
  2-subband model $\psi(\mathbf{r}) = \sum_{n=1,2} \psi_n(x)\phi_n(y)$

$$H_{\text{kin}} + H_{\text{conf}} \;\; \rightarrow \;\; H_0 = \sum_n \int dx \; \psi_n^\dagger \left(-\frac{\hbar^2 \partial^2}{2m} + \epsilon_n\right)\psi_n \qquad \epsilon_n = \hbar\Omega\left(n - \frac{1}{2}\right)$$

From 1D to quasi-1D: Non-interacting electrons

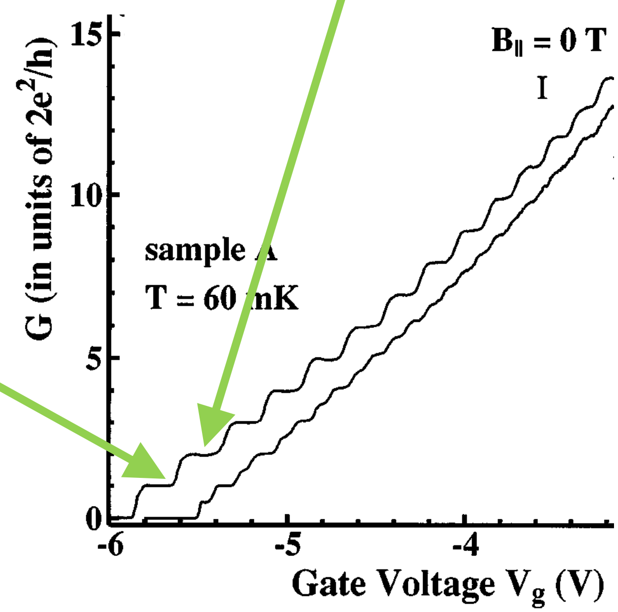


as the electron density is raised, the chemical potential moves above the second level and the system becomes quasi-1D



$\rightarrow$ **2** gapless excitation modes (= 2 independent 1D channels)

$$G = \frac{e^2}{h} \rightarrow \frac{2e^2}{h}$$



- at criticality:

$$\xi_1(k_x) = \frac{\hbar^2}{2m}k_x^2 + \epsilon_1 - \mu \simeq \hbar v_{F1} k_x$$

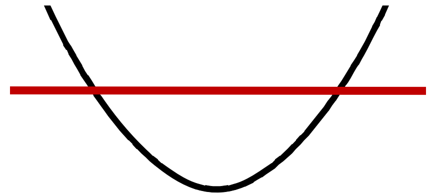
$$\xi_2(k_x) = \frac{\hbar^2}{2m}k_x^2 + \epsilon_2 - \mu = \frac{\hbar^2}{2m}k_x^2$$

Critical dynamics

lower subband:

$$\xi_1(k_x) = \frac{\hbar^2}{2m}k_x^2 + \epsilon_1 - \mu \simeq \hbar v_{F1} k_x$$

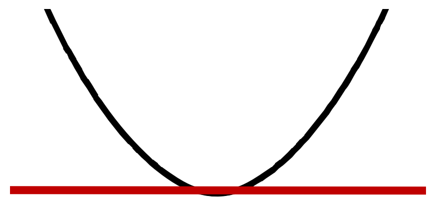
$\rightarrow\ \omega(k) \sim k$: dynamical exponent $z = 1$

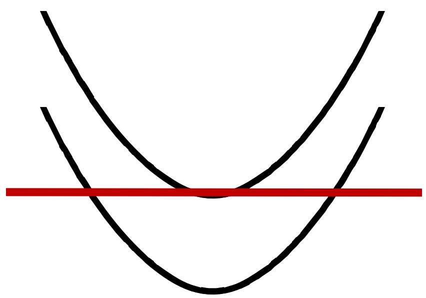


upper subband:

$$\xi_2(k_x) = \frac{\hbar^2}{2m}k_x^2 + \epsilon_2 - \mu = \frac{\hbar^2}{2m}k_x^2$$

$\rightarrow\ \omega(k) \sim k^2$: dynamical exponent $z = 2$





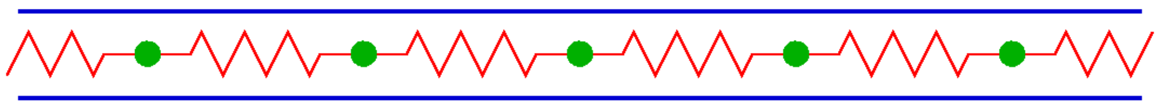
different coexisting dynamics!

Physics

From 1D to quasi-1D: Wigner crystal

$$E_{\text{Coul}} = \frac{e^2}{r} \sim e^2 n \quad \text{vs} \quad E_{\text{kin}} = \frac{\hbar^2 k_F^2}{2m} \sim \frac{1}{m}(\hbar n)^2$$

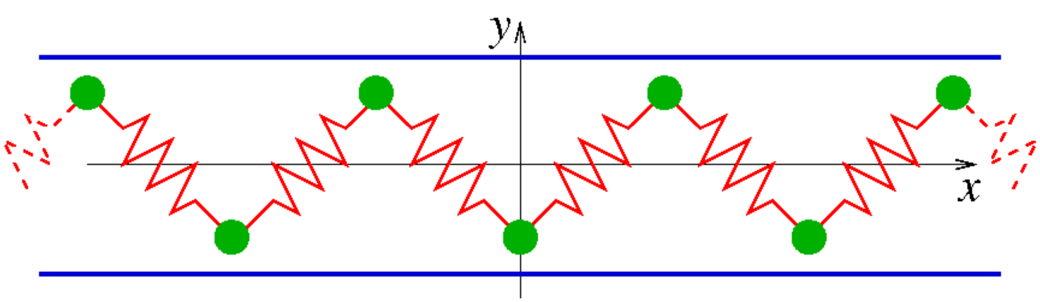
$n \to 0$: classical 1D Wigner crystal



parabolic confining potential $V_{\text{conf}} = \frac{1}{2} m \Omega^2 y^2$

$\to$ new length scale

$$r_0 = \left(\frac{2e^2}{m\Omega^2}\right)^{1/3}$$



crystal splits into 2 rows at

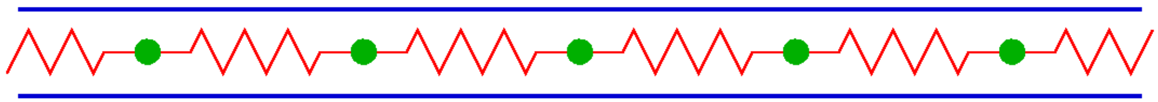
$$n \geq n_c \simeq \frac{0.78}{r_0}$$

Chaplik 1980, Piacente *et al*. 2004

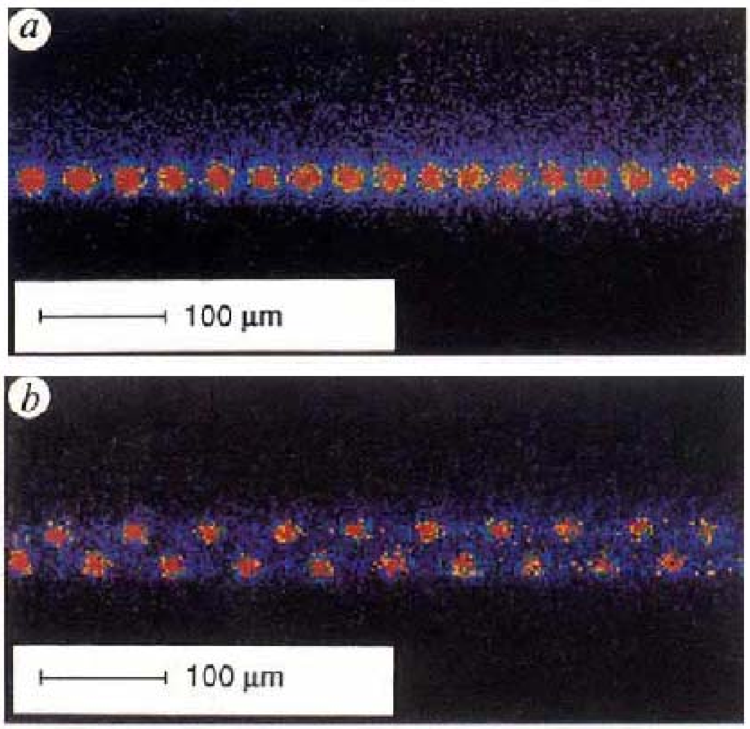
From 1D to quasi-1D: Wigner crystal

$$E_{\text{Coul}} = \frac{e^2}{r} \sim e^2 n \quad \text{vs} \quad E_{\text{kin}} = \frac{\hbar^2 k_F^2}{2m} \sim \frac{1}{m}(\hbar n)^2$$

$n \to 0$: classical 1D Wigner crystal



Birkl *et al.* 1992

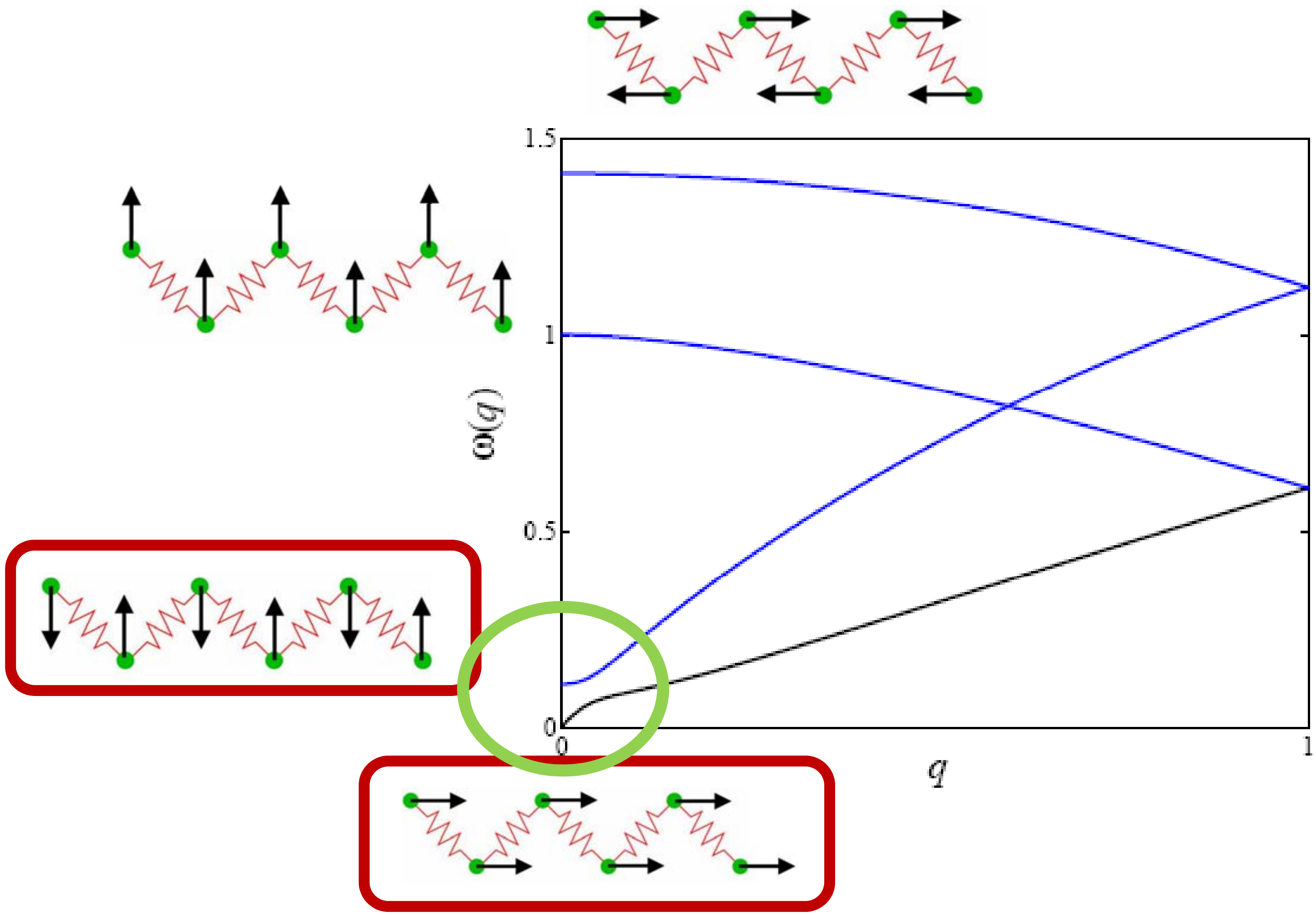


crystal splits into 2 rows at

$$n \geq n_c \simeq \frac{0.78}{r_0}$$

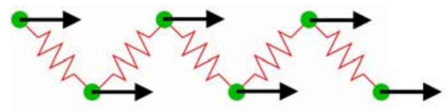
Chaplik 1980, Piacente *et al*. 2004

Phonons in a zigzag crystal

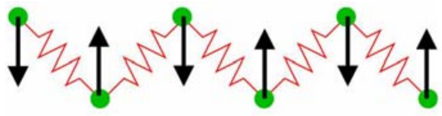


- only one gapless excitation mode (plasmon)
- a second gapless mode at the transition (soft mode)

At the transition …



$$\omega_{\parallel}(k) \sim k$$



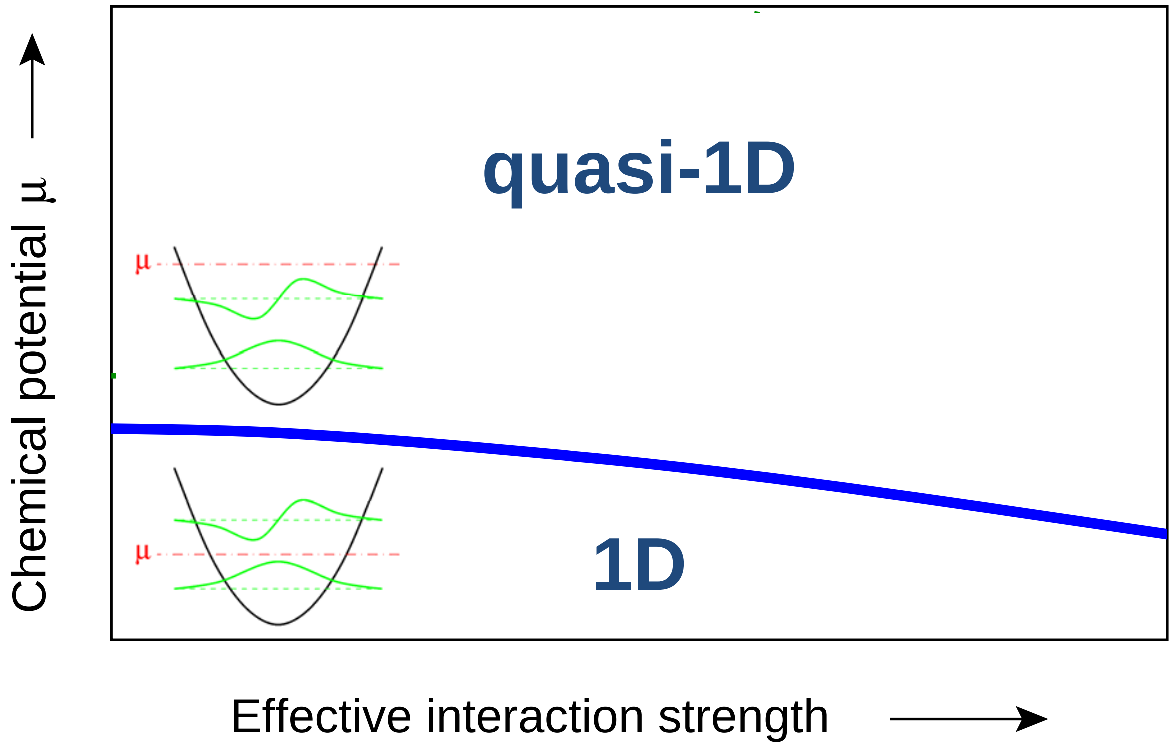
$$\omega_{\perp}(k) \sim k$$

- two linear phonon modes!

Phase diagram

non-
-interacting
electrons:

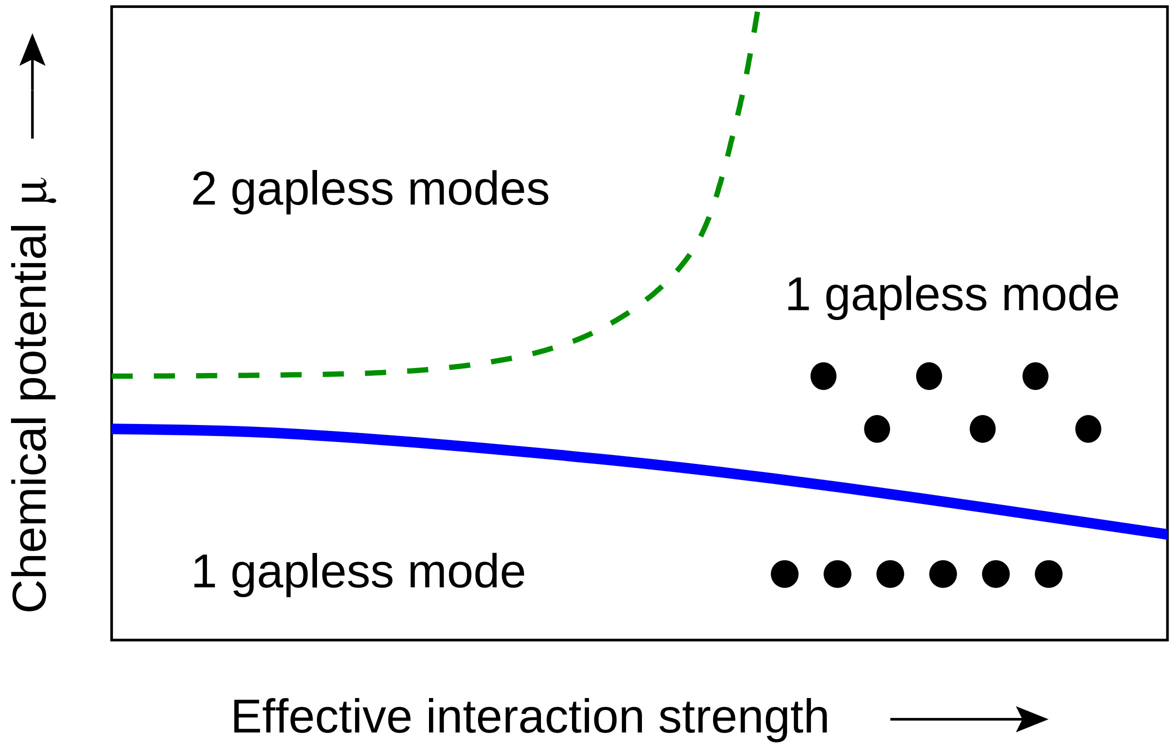
$\omega_1(k) \sim k$
$\omega_2(k) \sim k^2$



classical
Wigner
crystal:

$\omega_{\|}(k) \sim k$
$\omega_{\perp}(k) \sim k$

Phase diagram



JSM, K.A. Matveev, and A.I. Larkin, Phys. Rev. Lett. **98**, 126404 (2007)
JSM and K.A. Matveev, J. Phys.: Condens. Matter **21**, 023203 (2009)

The model: Interactions

- "not too strong" interactions:

  2-subband model $\psi(\mathbf{r}) = \sum_{n=1,2} \psi_n(x)\phi_n(y)$

$$H_{\text{kin}} + H_{\text{conf}} \;\; \rightarrow \;\; H_0 = \sum_n \int dx\, \psi_n^\dagger\left(-\frac{\hbar^2\partial^2}{2m} + \epsilon_n\right)\psi_n \qquad \epsilon_n = \hbar\Omega\left(n - \frac{1}{2}\right)$$

intra-subband & inter-subband interactions

$$V_{n_1 n_2 n_3 n_4}(|x - x'|) = \int dy\, dy'\; \phi_{n_1}(y)\phi_{n_2}(y)V(|\mathbf{r} - \mathbf{r}'|)\phi_{n_3}(y')\phi_{n_4}(y')$$

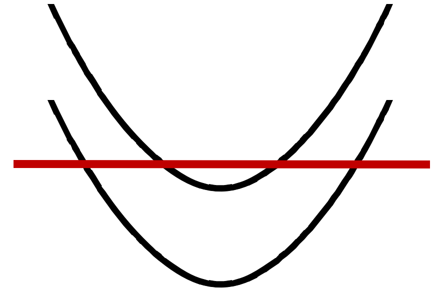
The interacting 2-subband model

$$V_{n_1 n_2 n_3 n_4}(|x - x'|) = \int dy\, dy'\ \phi_{n_1}(y)\phi_{n_2}(y)V(|\mathbf{r} - \mathbf{r}'|)\phi_{n_3}(y')\phi_{n_4}(y')$$

low-energy physics

$\rightarrow$ processes close to Fermi surface

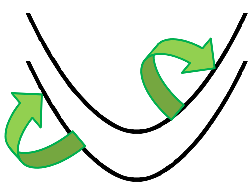
$\rightarrow$ momentum transfer restricted



- intra-subband interactions: $V_{nnnn}(k \approx 0)$ $V_{nnnn}(k \approx 2k_{Fn})$

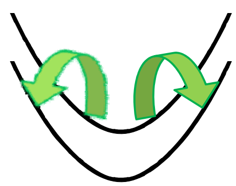
  $g_n = V_{nnnn}(0) - V_{nnnn}(2k_{Fn})$

- inter-subband interactions:



$V_{1122}(k \approx 0)$

$V_{1221}(k \approx k_{F1} + k_{F2})$



$V_{1212}(k \approx k_{F1} - k_{F2})$

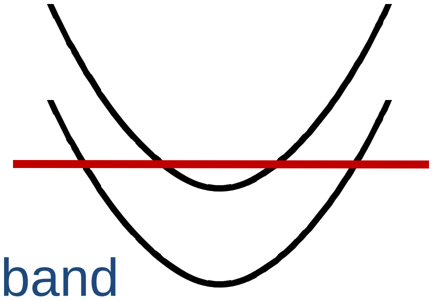
$V_{1212}(k \approx k_{F1} + k_{F2})$

$g_x = V_{1122}(0) - V_{1221}(k_{F1} + k_{F2})$

$g_t = V_{1212}(k_{F1} - k_{F2}) - V_{1212}(k_{F1} + k_{F2})$

THE OHIO STATE UNIVERSITY Physics

Bosons & fermions



- bosonize electrons in the lower subband
  while keeping a fermionic description for the upper subband
  Balents, PRB **61**, 4429 ('00)

$$H_1 = \frac{\hbar v_{F1}}{2\pi} \int dx \left((\partial\theta_1)^2 + \frac{1}{K^2} (\partial\phi_1)^2 \right)$$

Lorentz invariant $\rightarrow$ dynamical exponent $z = 1$

$$H_2 = -\frac{\hbar^2}{2m} \int dx\, \psi_2^\dagger \partial^2 \psi_2 - r \int dx\, \psi_2^\dagger \psi_2$$

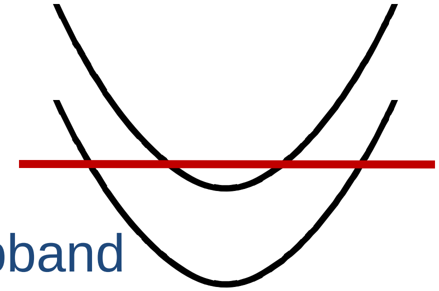
Galilean invariant $\rightarrow$ dynamical exponent $z = 2$

NOTE: $g_2 = V_{2222}(0) - V_{2222}(2k_{F2}) \propto k_{F2}^{\,2} \rightarrow 0$

where K Luttinger liquid parameter (lower subband)

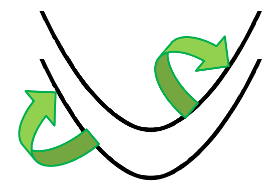
$r = \mu - \mu_c$

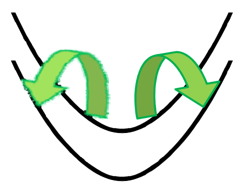
Bosons & fermions + Interactions



- bosonize electrons in the lower subband
  while keeping a fermionic description for the upper subband

$$
\begin{aligned}
H_1 &= \frac{\hbar v_{F1}}{2\pi}\int dx\,\left((\partial\theta_1)^2+\frac{1}{K^2}(\partial\phi_1)^2\right)\\
H_2 &= -\frac{\hbar^2}{2m}\int dx\,\psi_2^\dagger\partial^2\psi_2 - r\int dx\,\psi_2^\dagger\psi_2\\
H_{12} &= -\frac{1}{\pi}\sum_q(\partial\phi_1)(q)V_{12}(q)n_2(-q)\\
&\quad+\gamma_t\int dx\,\left[(\psi_2^\dagger\partial\psi_2^\dagger-\partial\psi_2^\dagger\psi_2^\dagger)e^{-2i\theta_1}+\text{h.c.}\right]
\end{aligned}
$$





where K Luttinger liquid parameter (lower subband)

$r = \mu - \mu_c$

and $\gamma_t \sim e^2$

Unitary transformation

- use unitary transformation to remove part of the Hamiltonian H_{12}

$$U = \exp\left[-\frac{iK^2}{\pi\hbar v_{F1}}\int dx\, dy\, \theta_1(x)V(x-y)n_2(y)\right]$$

Balents, PRB **61**, 4429 (‘00)

new fields: $\partial\phi_1 \to \partial\phi_c = \partial\phi_1 + \frac{K^2 g_x}{v_{F1}}\psi_2^\dagger\psi_2$ $\quad\psi_2 \to \psi_s = \psi_2 e^{-i\frac{K^2 g_x}{\pi v_{F1}}}$

effective Hamiltonian:

$$H_c = \frac{\hbar v}{2\pi}\int dx\left(K(\partial\theta_c)^2 + \frac{1}{K}(\partial\phi_c)^2\right)$$ plasmons

$$H_s = -\frac{\hbar^2}{2m}\int dx\, \psi_s^\dagger\partial^2\psi_s - r\int dx\, \psi_s^\dagger\psi_s$$ dressed fermions

$$H_{cs} = \gamma_t\int dx\left[(\psi_s^\dagger\partial\psi_s^\dagger - \partial\psi_s^\dagger\psi_s^\dagger)e^{-2i\kappa\theta_c(x)} + \text{h.c.}\right]$$

where $\kappa = 1 - K^2 g_x/(\pi v_{F1}) \simeq K^2$

JSM, Matveev, Larkin, PRL (2007)

Weak interactions

$K^2 g_x/(\pi\hbar v_{F1}) \ll 1$:

pair-tunneling irrelevant
$\Rightarrow$ as in the non-interacting case, but in terms of dressed particles:

$$H_c = \frac{\hbar v}{2\pi}\int dx\left(K(\partial\theta_c)^2 + \frac{1}{K}(\partial\phi_c)^2\right)$$

Lorentz invariant $\rightarrow$ dynamical exponent $z = 1$

$$H_s = -\frac{\hbar^2}{2m}\int dx\, \psi_s^\dagger \partial^2 \psi_s - r\int dx\, \psi_s^\dagger \psi_s$$

Galilean invariant $\rightarrow$ dynamical exponent $z = 2$

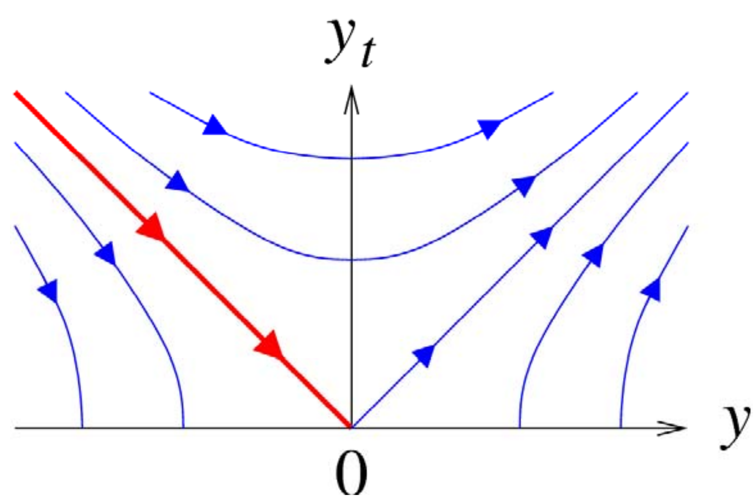
pair-tunneling not important for critical properties,
BUT: pair-tunneling dangerously irrelevant …

Weak interactions: Away from criticality

close to the transition $(v_{F2} \ll v_{F1})$:

$$y = -\frac{1}{2\pi\hbar}\left(\frac{g_1 - 4g_x}{v_{F1}} + \frac{g_2}{v_{F2}}\right)$$

$$y_t = \frac{g_t}{\pi\hbar\sqrt{2v_{F1}v_{F2}}}$$



renormalization of coupling constants:

$$y' = y_t^2 \qquad y_t' = y y_t$$

Muttalib & Emery 1986, Ledermann & Le Hur 2000

NOTE: $g_t^{(0)} \propto k_{F2}$ $\qquad g_2^{(0)} \propto k_{F2}{}^2$ $\qquad g_1^{(0)} \simeq g_x^{(0)}$

$\Rightarrow$ gap $\quad \Delta \propto (\mu - \mu_c)^\alpha \qquad (\alpha \sim \frac{\hbar v_{F1}}{e^2})$

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Strong interactions

$\kappa \approx K^2 \to 0$:

$$H_c = \frac{\hbar v}{2\pi} \int dx \left(K(\partial\theta_c)^2 + \frac{1}{K}(\partial\phi_c)^2 \right)$$

$$H_s = \int dx \left\{ -\frac{\hbar^2}{2m}\psi_s^\dagger \partial^2 \psi_s - r\psi_s^\dagger\psi_s + \frac{u}{2}\left[(\psi_s^\dagger \partial \psi_s^\dagger - \partial\psi_s^\dagger \psi_s^\dagger) + \text{h.c.} \right] \right\}$$

decoupling of bosons & fermions!

- pair-tunneling relevant
- transition in the Ising universality class:

  dynamical critical exponent $z = 1$

  gap $\Delta \sim |\mu - \mu_c|$

- effect of weak coupling between the modes? $(\psi_s^\dagger \partial \psi_s^\dagger - \partial\psi_s^\dagger \psi_s^\dagger) e^{-2i\kappa\theta_c(x)}$

Unitary transformation II

- use unitary transformation $U = \exp\left[-\frac{i}{\hbar}\int dx\,\theta_1 n_2\right]$

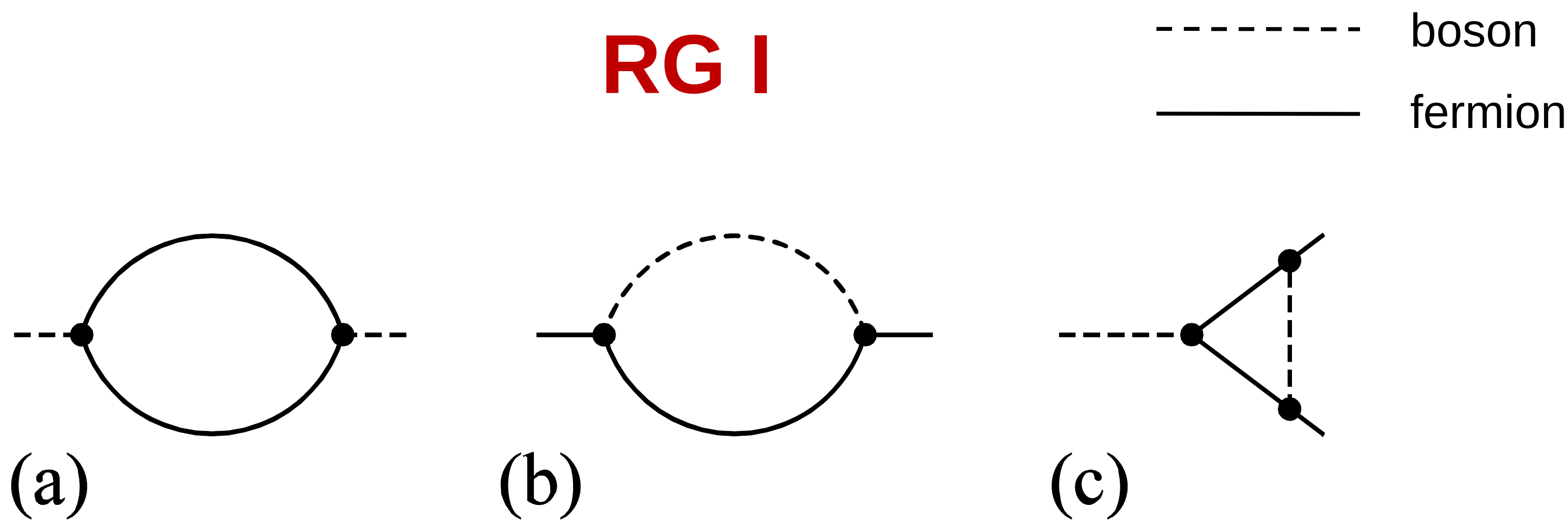
effective Hamiltonian:

$$
\begin{aligned}
H_c &= \frac{\hbar v}{2\pi}\int dx\,\left(K(\partial\theta_c)^2 + \frac{1}{K}(\partial\phi_c)^2\right) \\
H_s &= -r\int dx\,\psi_s^\dagger\psi_s + \frac{u}{2}\int dx\,\left[\left(\psi_s^\dagger\partial\psi_s^\dagger - \partial\psi_s^\dagger\psi_s^\dagger\right) + \mathrm{h.c.}\right] \\
H'_{cs} &= -\frac{1}{\pi}\lambda\int dx\,\partial\phi_c\psi_s^\dagger\psi_s
\end{aligned}
$$

NOTE: H'_{cs} breaks Lorentz invariance

RG …

RG I



RG I

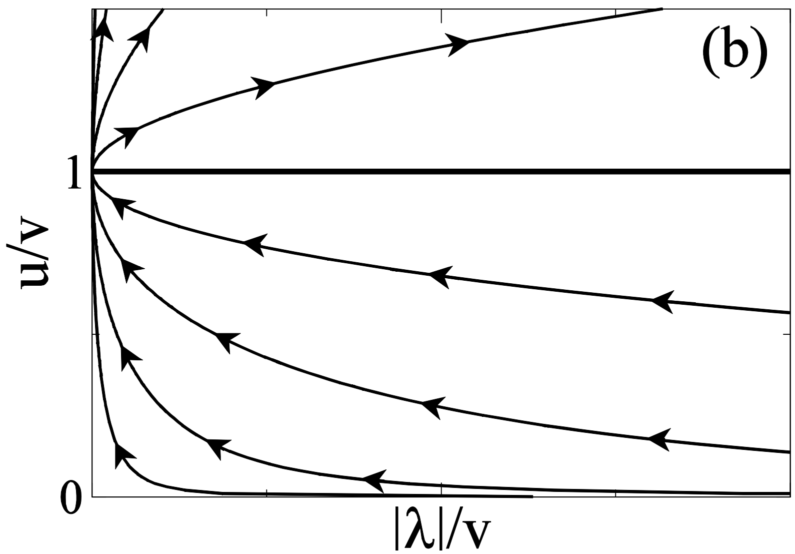
introduce $x = u/v, \;\; g = \lambda/v$

RG equations:

$$x' = \frac{K^2}{4\pi^2} g^2 \left[1 - \frac{4x}{(1+x)^2} \right]$$

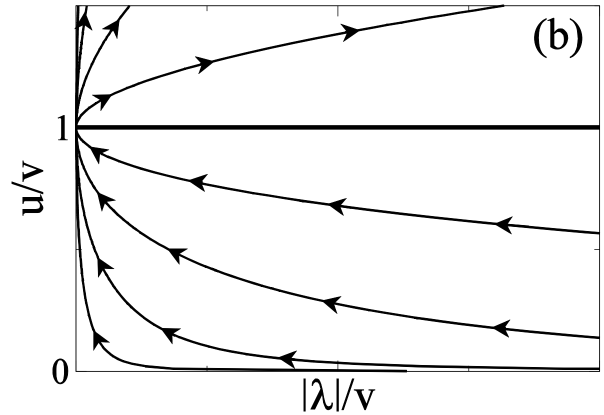
$$g' = -\frac{K^2}{2\pi^2} g^3 \left[\frac{1}{x(1+x)} + \frac{1}{(x+1)^2} - \frac{3}{4x} \right]$$

flow diagram:



here
$u \ll v$

RG II

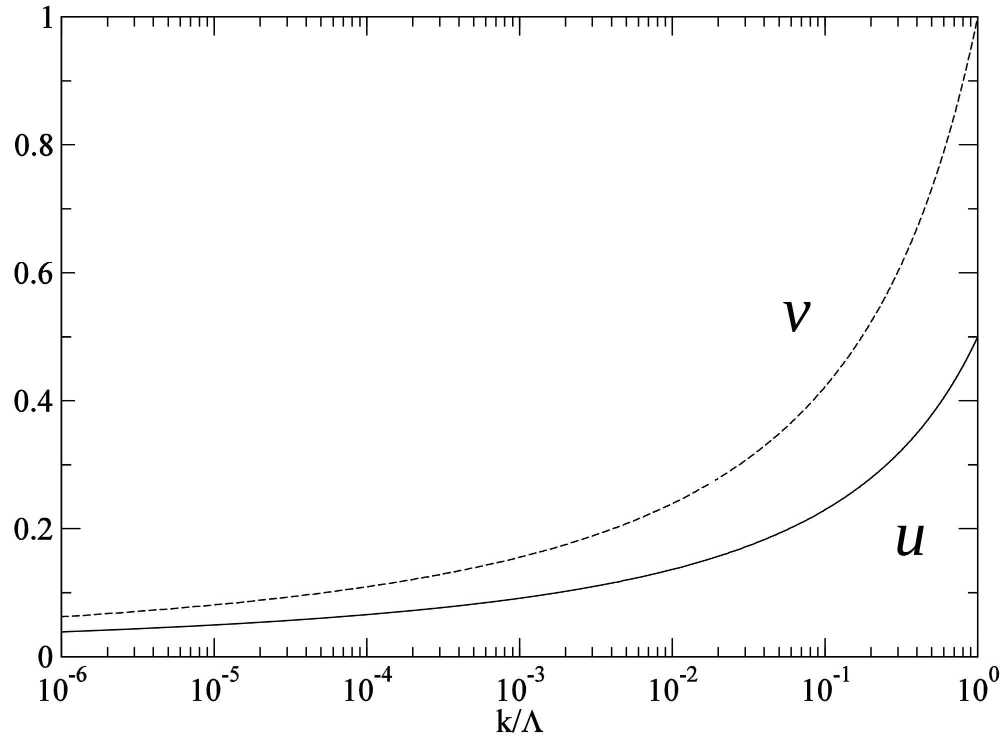


- λ marginally irrelevant
- velocities approach $u \to v$
- fixed point: conformal field theory

  with enhanced SU(2) symmetry & central charge $c = 3/2$

BUT: $v' = -\frac{K^2}{4\pi^2} g^2 \frac{1}{x} v$

velocities flow to 0!



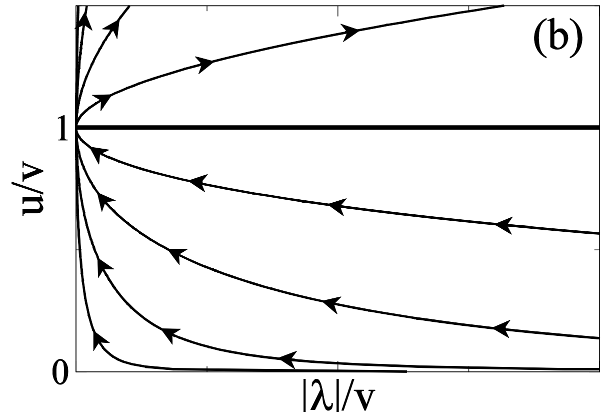
Consequences?

- divergent specific heat coefficient

$$\gamma \approx \frac{\pi}{6}\left(\frac{2}{v} + \frac{1}{u}\right)$$

M. Sitte, A. Rosch, JSM, K.A. Matveev, and M. Garst, PRL **102**, 176404 (2009)

RG II



- λ marginally irrelevant
- velocities approach $u \to v$
- fixed point: conformal field theory

  with enhanced SU(2) symmetry & central charge $c = 3/2$

BUT: $v' = -\dfrac{K^2}{4\pi^2} g^2 \dfrac{1}{x} v$

velocities flow to 0!

- dispersion $\epsilon(k) \sim k \exp\left[-A_\epsilon \left(\ln \frac{1}{k}\right)^{1/5}\right]$

  (dynamical exponent marginally larger than 1)

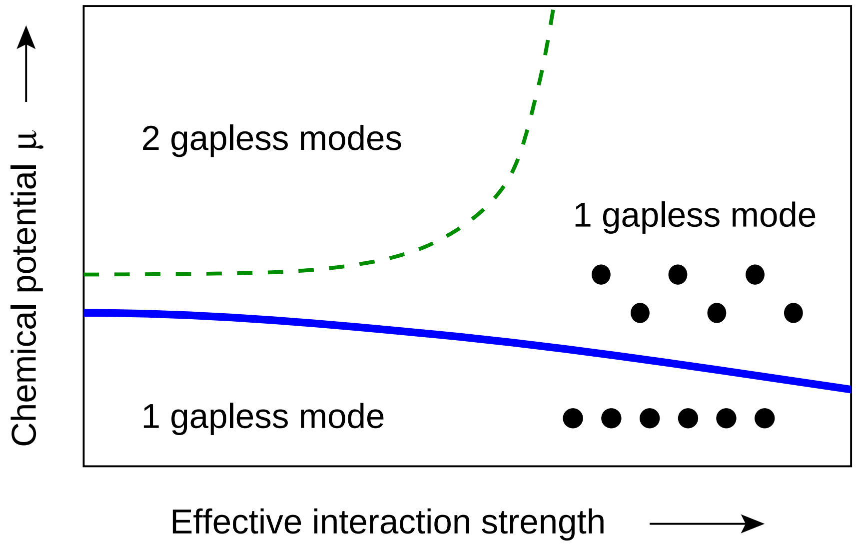
- at finite r:

  gap $\Delta \sim |\mu - \mu_c| \exp\left[-A_\Delta \left(\ln |\mu - \mu_c|\right)^{1/5}\right]$

M. Sitte, A. Rosch, JSM, K.A. Matveev, and M. Garst, PRL **102**, 176404 (2009)

Conclusions

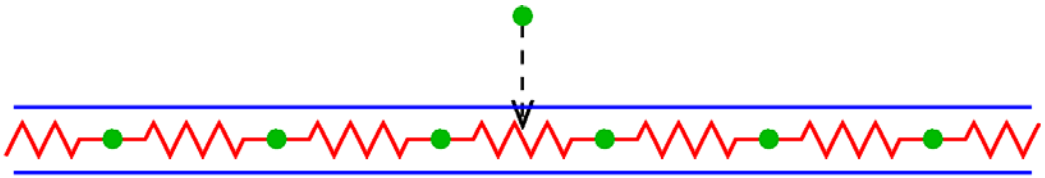
- weak interactions:
  multiscale quantum critical point
  with coexisting dynamics
  $z = 1$ and $z = 2$
- strong interactions:
  emergence of
  single dynamical scale



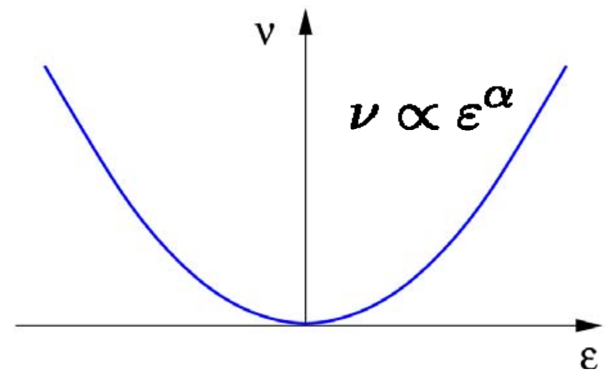
- fixed-point Lorentz invariant CFT with $c = 3/2$
- BUT: vanishing velocity due to marginal irrelevant residual interaction

- new gapped excitation mode appears above the transition
- gap can be observed in the tunneling density of states

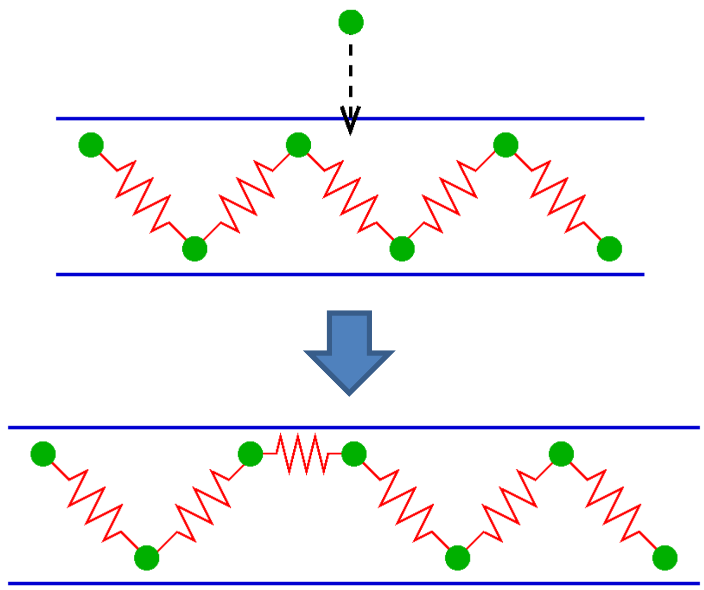
Density of states near the zigzag transition



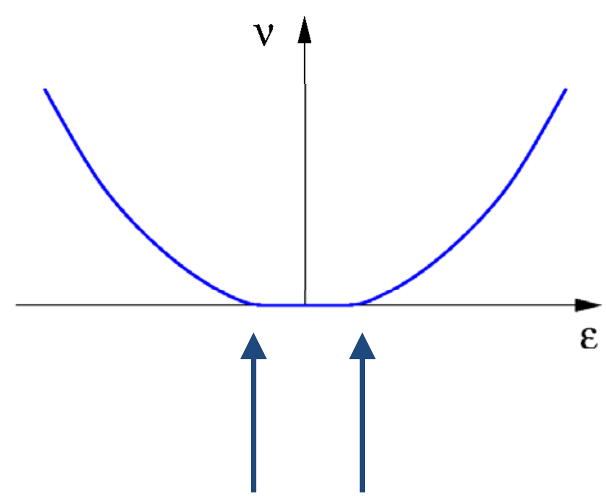
1D Wigner crystal $\leftrightarrow$ Luttinger liquid



zigzag:



defect!

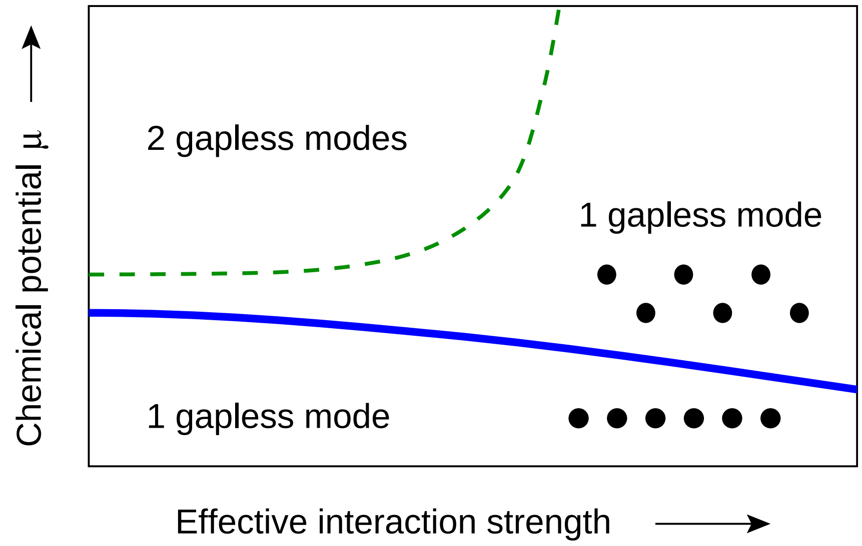


finite energy gap

Starykh *et al.* 2000

Conclusions

- weak interactions:
  multiscale quantum critical point
  with coexisting dynamics
  $z = 1$ and $z = 2$
- strong interactions:
  emergence of
  single dynamical scale



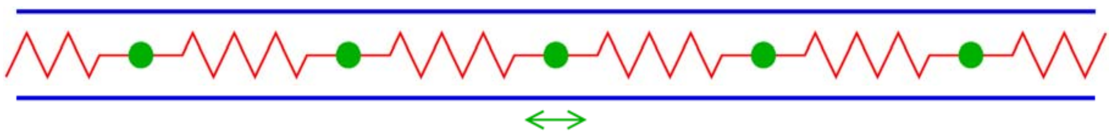
- fixed-point Lorentz invariant CFT with $c = 3/2$
- BUT: vanishing velocity due to marginal irrelevant residual interaction

- new gapped excitation mode appears above the transition
- gap can be observed in the tunneling density of states

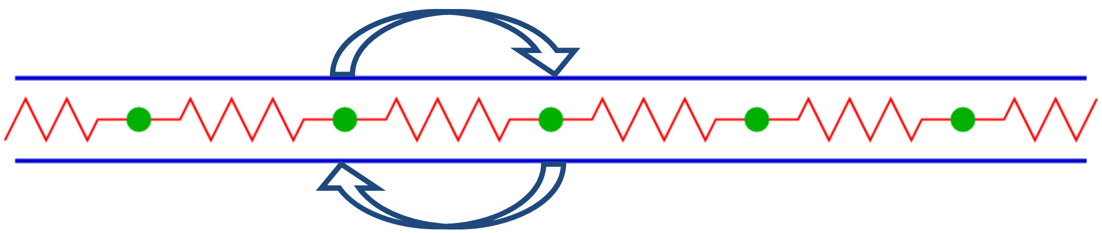
Open questions:

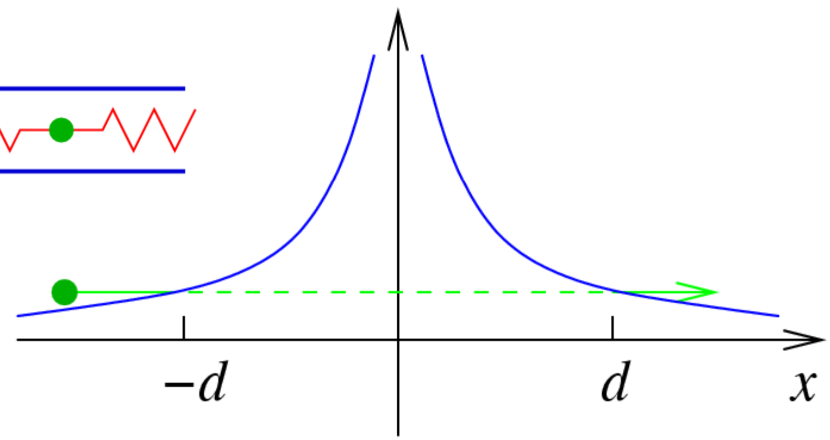
- spins (gap at weak interactions is due to the Pauli principle …)

Spin interactions in a Wigner crystal



- to a first approximation, spins do not interact ...
- BUT:
  weak tunneling through Coulomb barrier

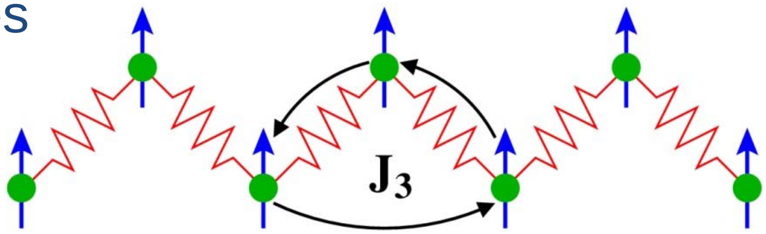




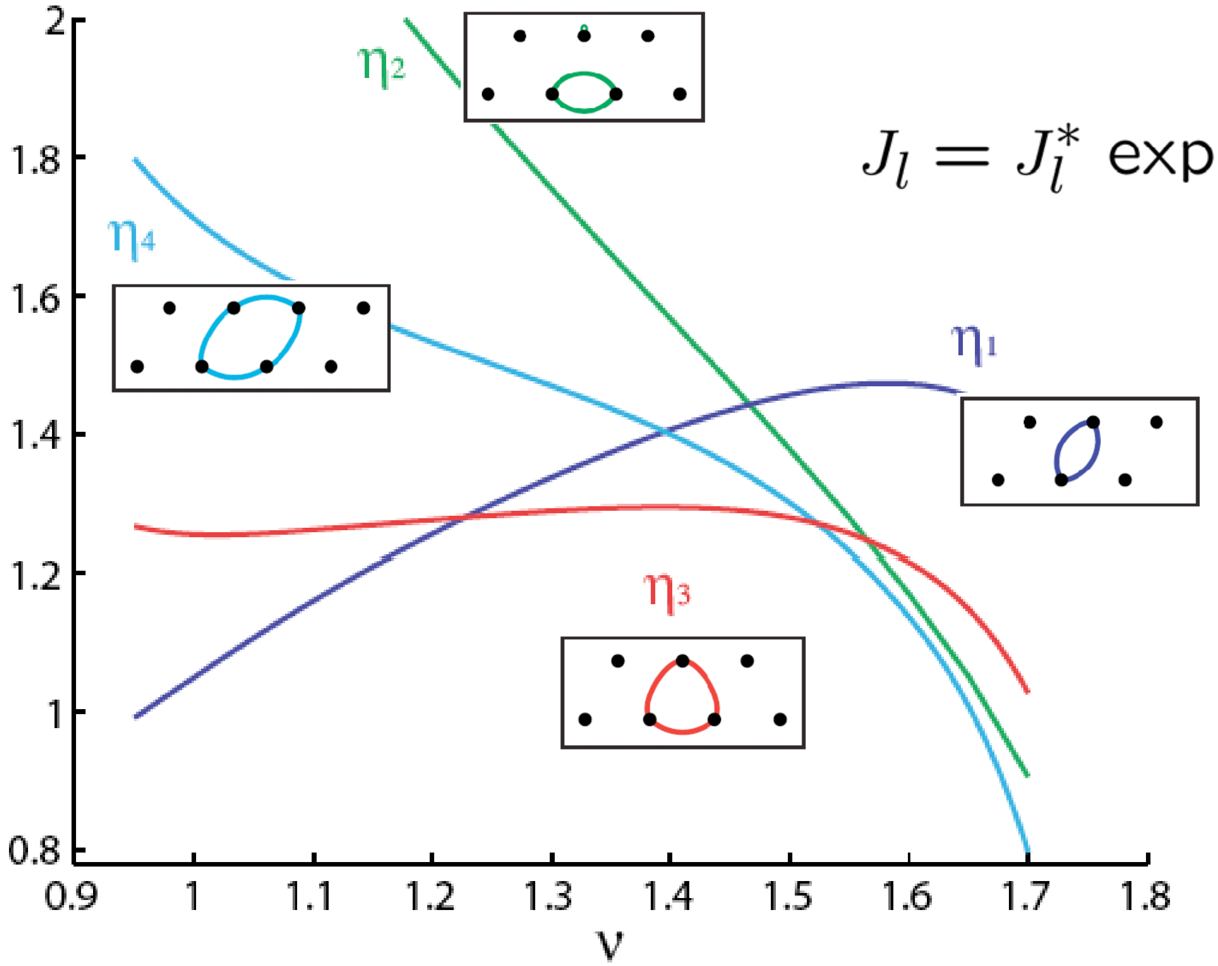
$\Rightarrow$ exponentially small
exchange constants J

Spin properties of the zigzag crystal

- spin interactions due to ring exchanges



Numerical results

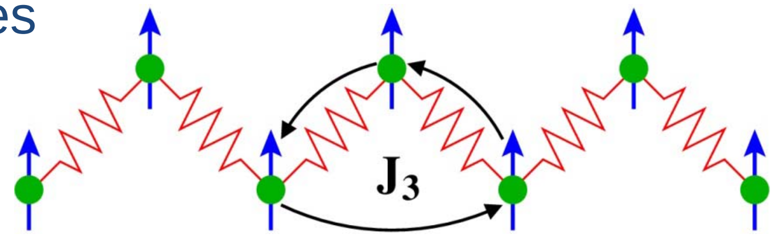


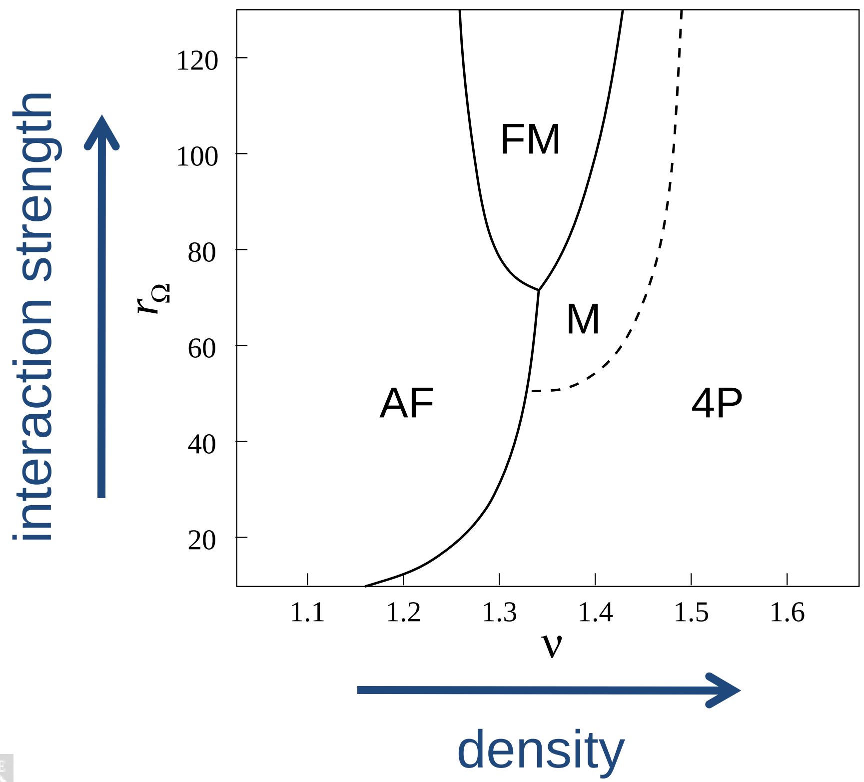
$$J_l = J_l^* \exp\left[-\eta_l \sqrt{r_\Omega}\right]$$

dominant exchange: $J_1 \rightarrow J_3 \rightarrow J_4$

Spin properties of the zigzag crystal

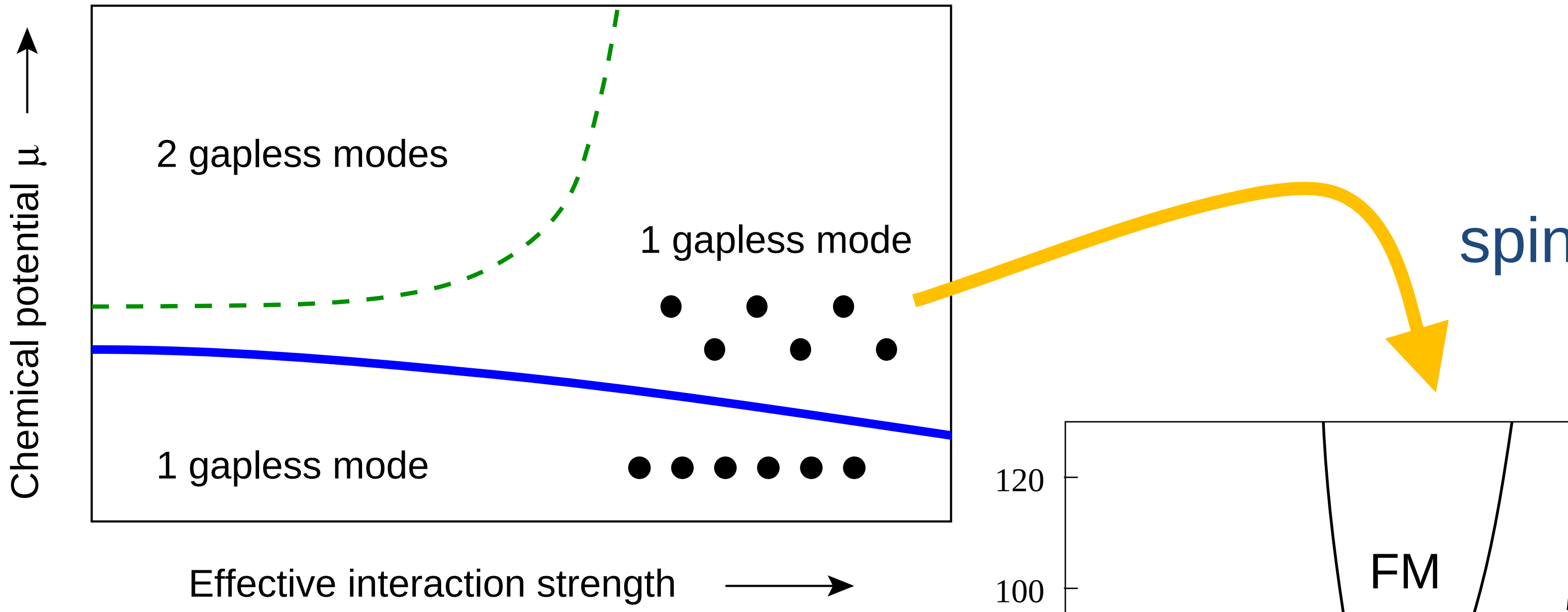
- spin interactions due to ring exchanges

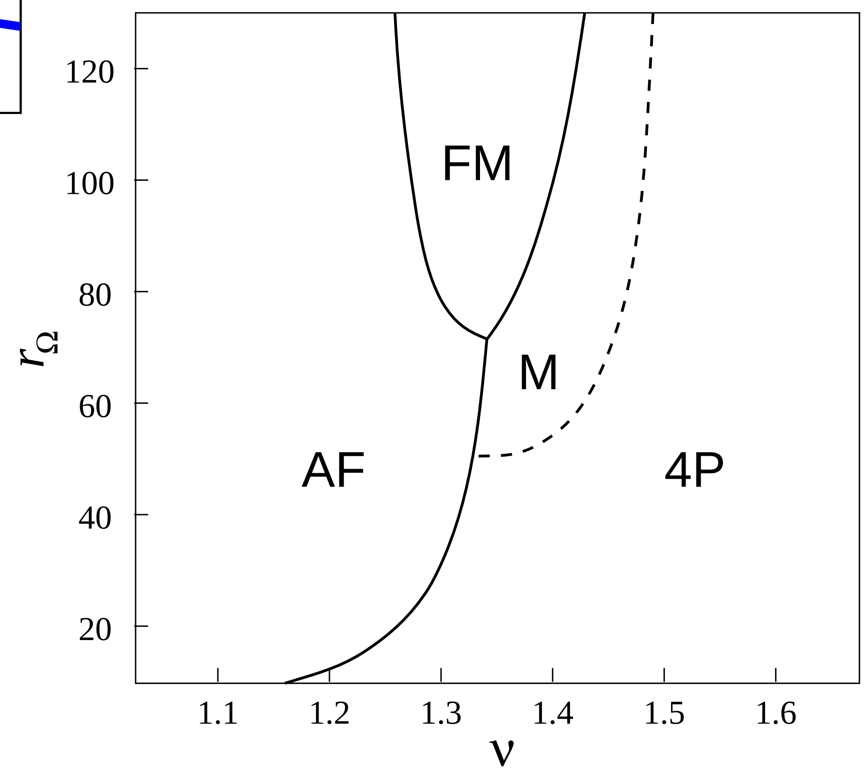


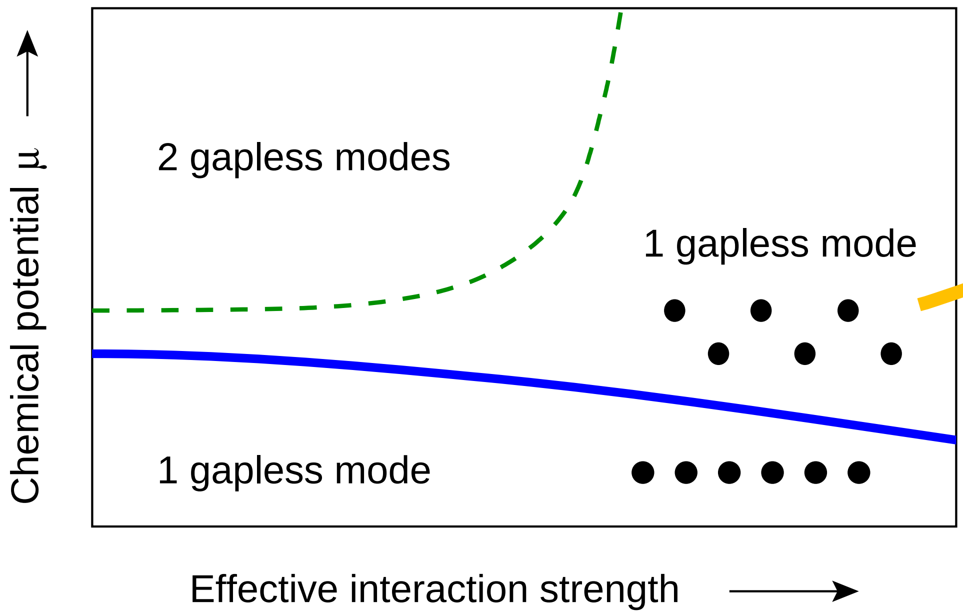


Klironomos, JSM, Matveev
Europhys. Lett. **74**, 679 (2006)

Klironomos, JSM, Hikihara, Matveev
Phys. Rev. B **76**, 75302 (2007)


Chemical potential μ
2 gapless modes
1 gapless mode
1 gapless mode
Effective interaction strength
spin


120
100
80
60
40
20
r_Ω
FM
M
AF
4P
1.1
1.2
1.3
1.4
1.5
1.6
ν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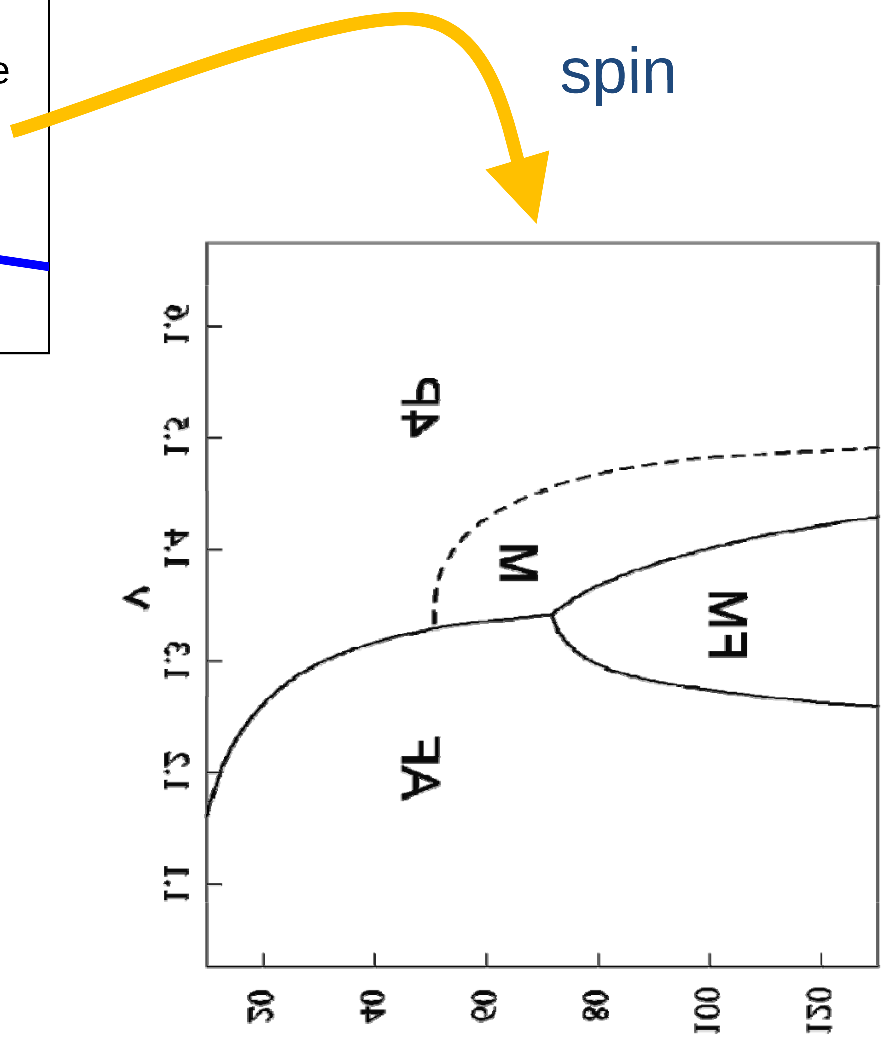
spin

The End …