

ATTRACTIVE FERMION GASES AT INFINITE

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BCS '57 Fermions $\uparrow\downarrow$ with density $n = k_F^3/3\pi^2$ and

attractive two-particle interaction $V_{\uparrow\downarrow}(\mathbf{x}) = \bar{g} \cdot \delta(\mathbf{x})$

pairs form and condense at $T_c \sim \exp\left[-\frac{1}{|\bar{g}|N(0)}\right] \ll T_F$

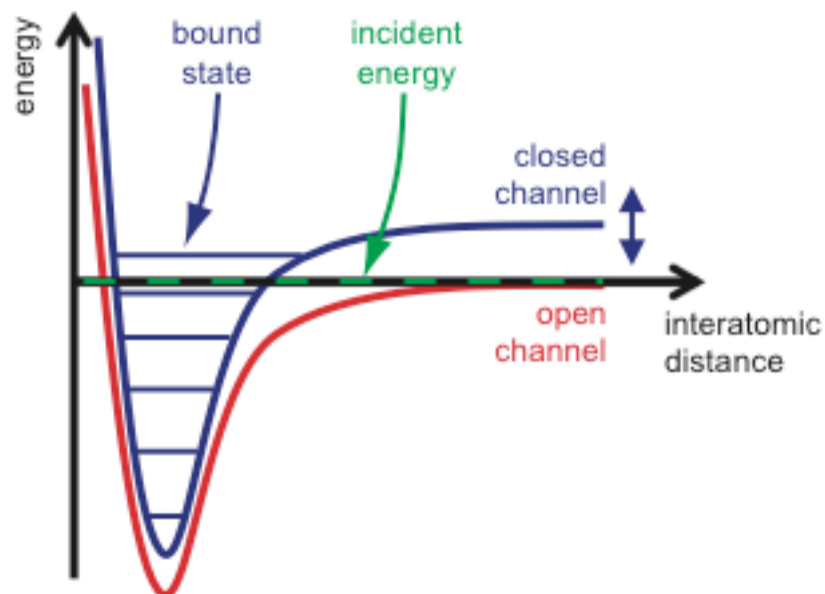
what happens at infinite coupling $g = \infty$?

FESHBACH-RESONANCES

closed channel bound state

couples resonantly

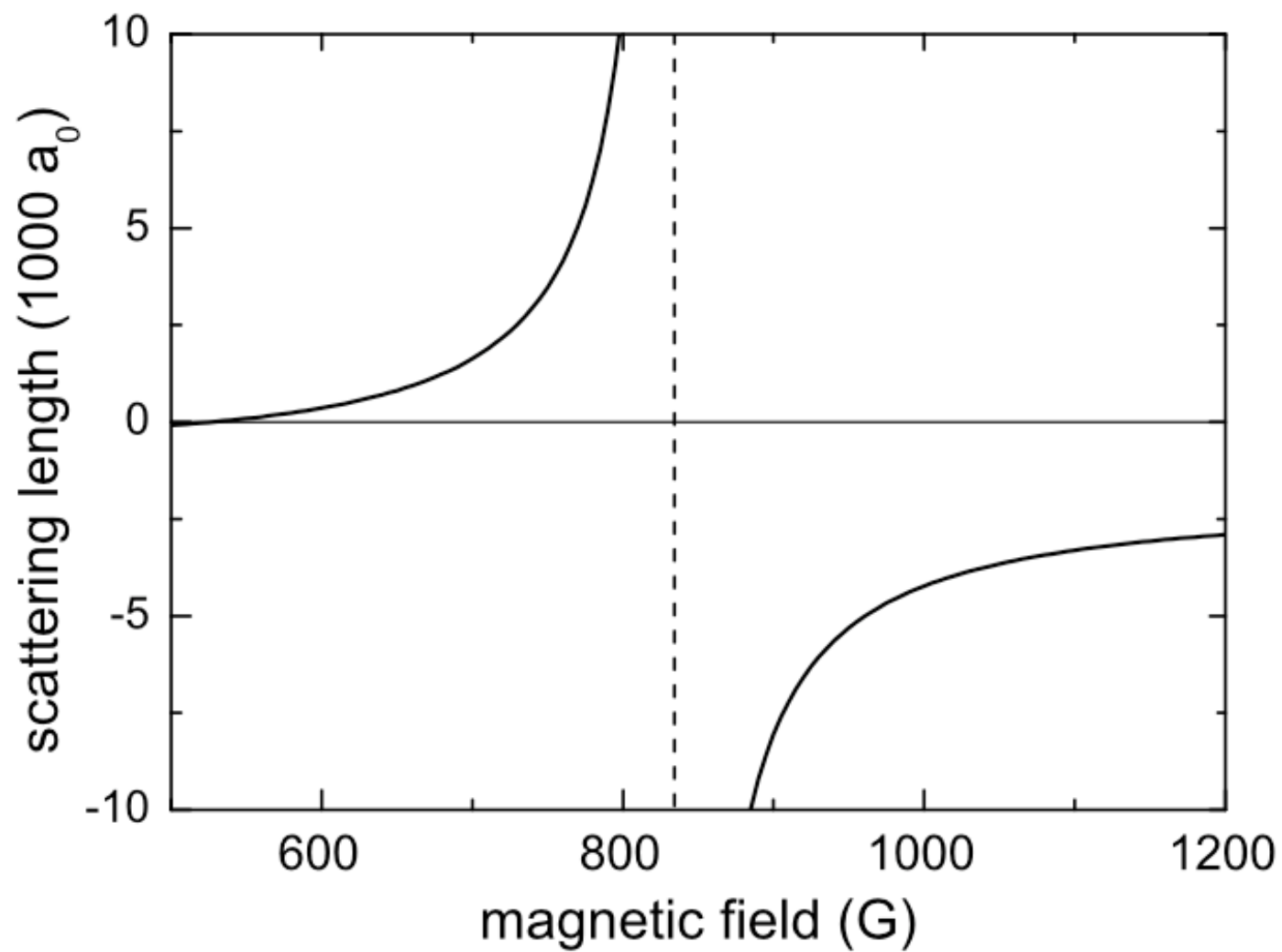
$$a_s = a_{bg} \left(1 - \frac{\Delta B}{B - B_0} \right)$$



scatt. ampl. $f(k) = \frac{-1}{1/a_s + r^*k^2 + ik} \rightarrow \frac{i}{k}$ at $a_s = \infty$

broad resonance if effective range $r^* \ll \lambda_F$

scattering length in ${}^6\text{Li}$ $gN(0) \rightarrow 2k_F a/\pi$



Outline

- I) **Thermodynamics at Unitarity**
 - II) **Spectral functions and rf-spectroscopy**
 - III) **Imbalanced Fermi gases**
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I) Thermodynamics at Unitarity ($a = \infty$)

$\mathbf{x} \rightarrow \lambda \mathbf{x}$ gives $H \rightarrow H/\lambda^2$ **scale invariance**

implies conformal invariance \rightarrow **Tr T = 0** \rightarrow

pressure $p = 2u/3$ **Ho '04** bulk viscosity $\zeta = 0$ **Son '07**

ground state $p(\infty) = \xi \cdot p_F^{(0)}$ **Bertsch-parameter** ξ

determines cloud size in a trap $R_{TF} = R_{TF}^{(0)} \cdot \xi^{1/4}$

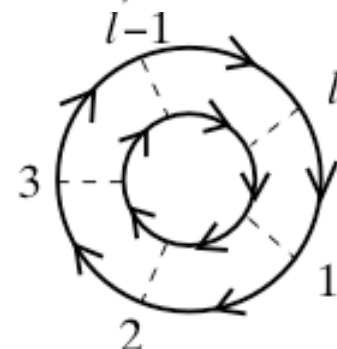
Many-body theory pseudopotential $V_{\uparrow\downarrow}(\mathbf{x}) = \bar{g} \delta(\mathbf{x})$

Luttinger/Ward '60 $\Omega = -T \ln Z = \Omega[\hat{G}]$

$$\Omega[\hat{G}] = \beta^{-1} \left(-\frac{1}{2} \text{Tr} \{ -\ln \hat{G} + [\hat{G}_0^{-1} \hat{G} - 1] \} - \Phi[\hat{G}] \right)$$

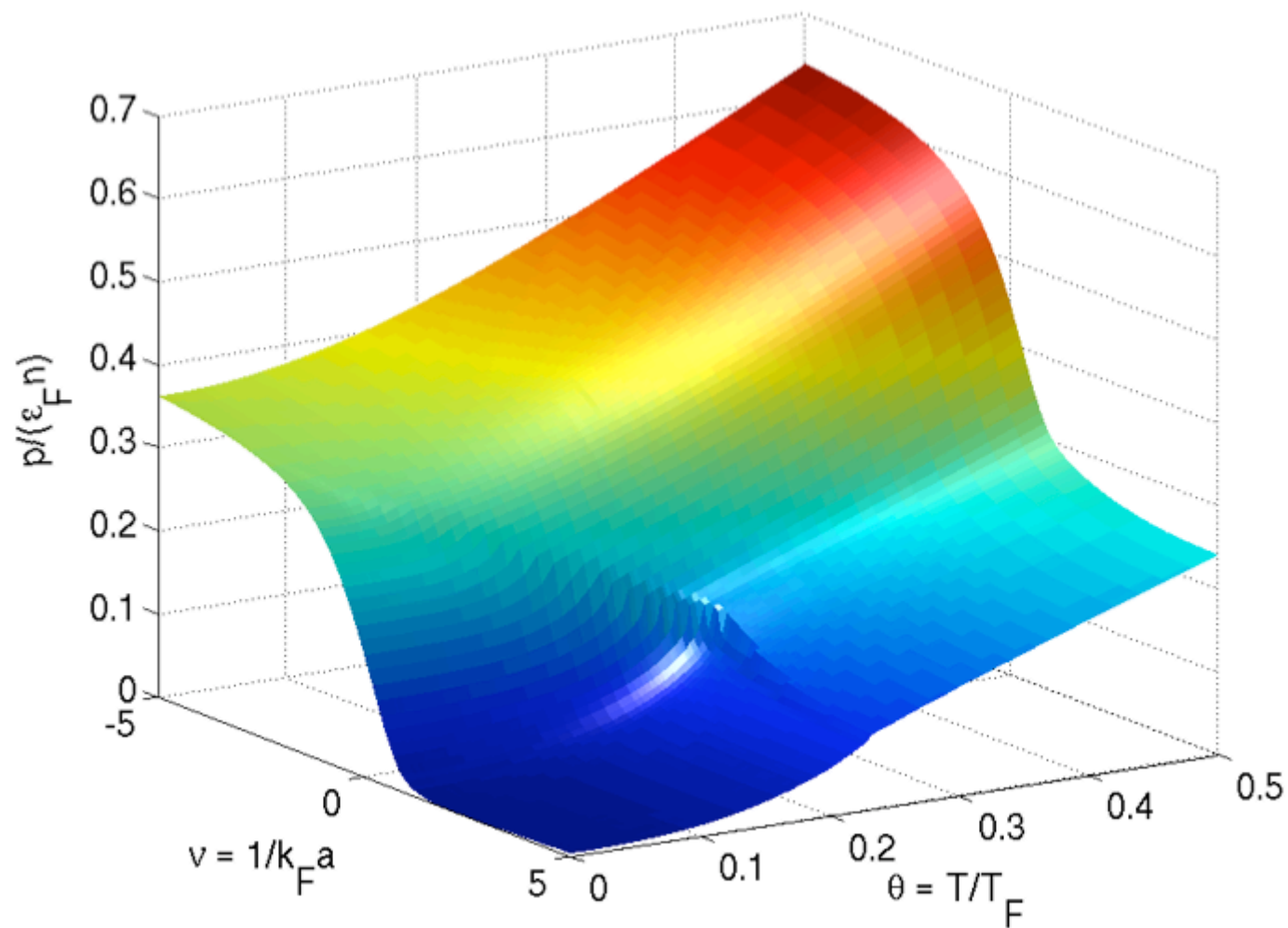
Ladder-approximation

$$\Phi[\hat{G}] = \sum_{l=0}^{\infty}$$

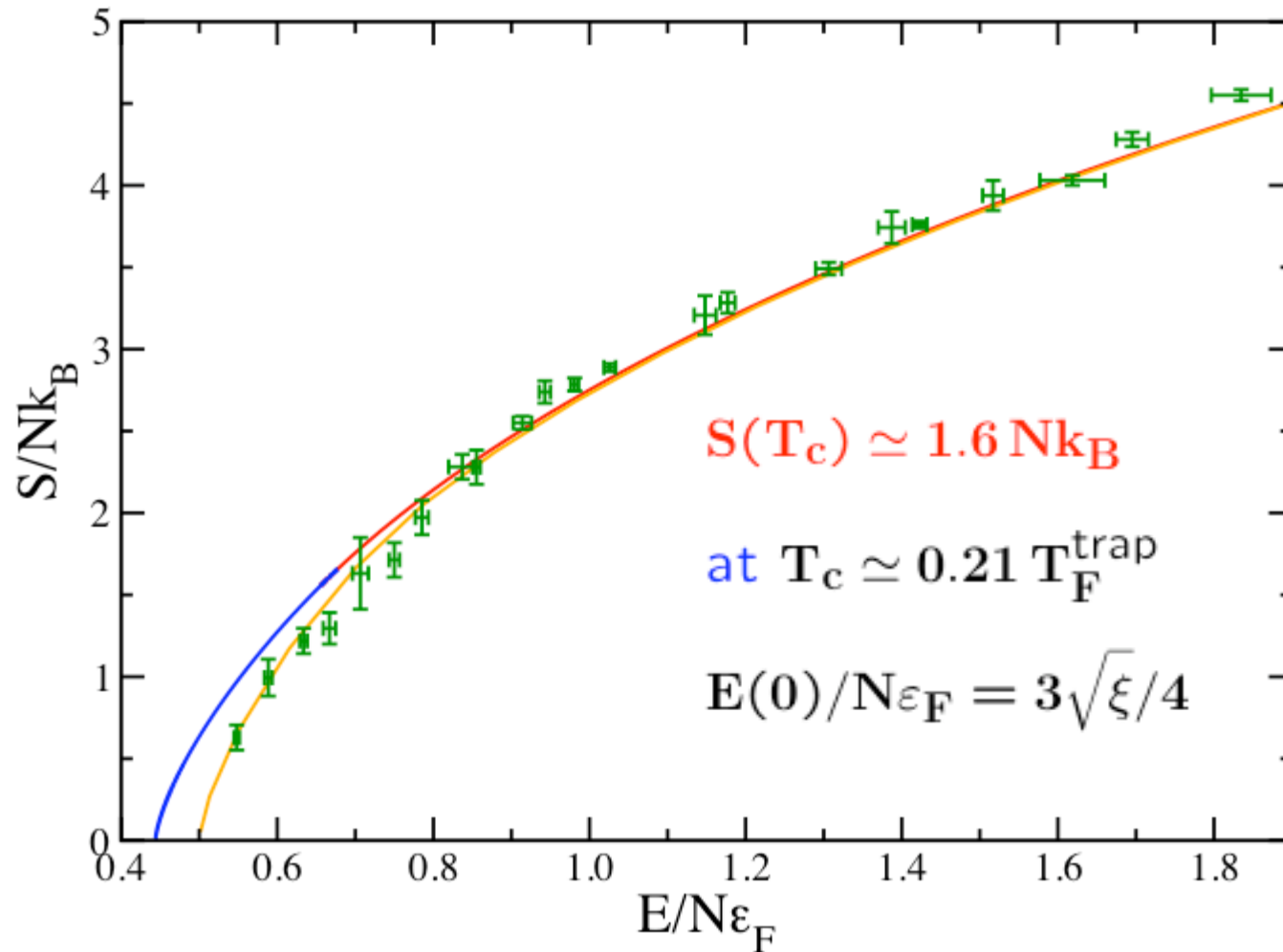


$\delta\Omega[\hat{G}]/\delta\hat{G} = 0$ variational principle for functions !

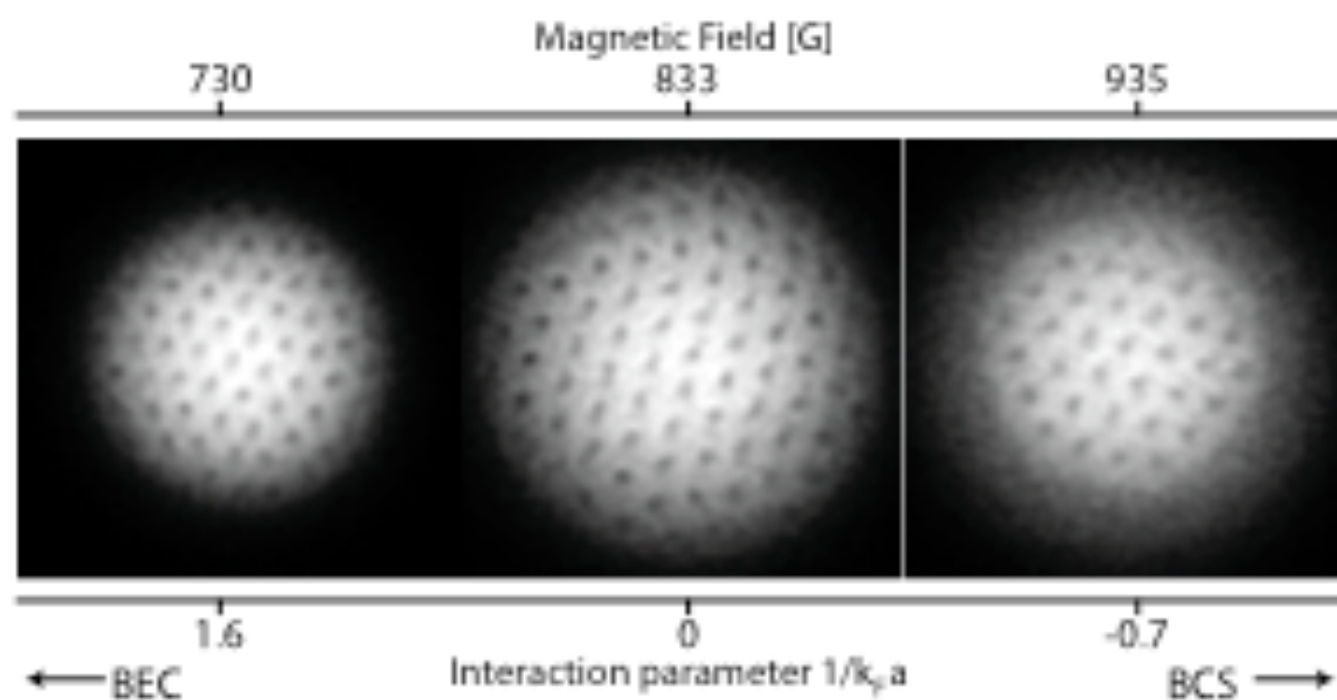
pressure as a function of T/T_F and $1/k_F a$ ($\xi = 0.36$)



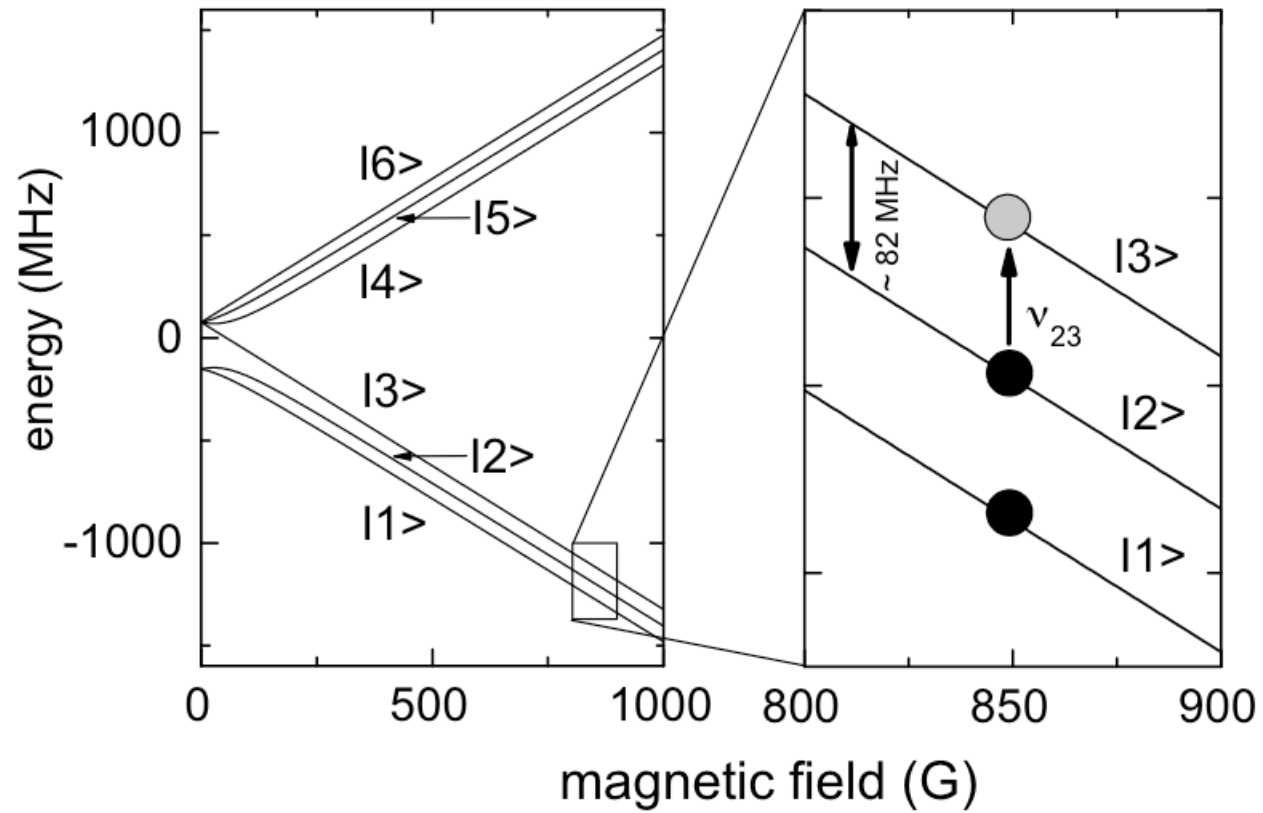
Unitary gas entropy in a trap Thomas '07+'09



Superfluidity from vortex lattice Zwierlein et.al. '05



II) Spectral functions and rf-spectroscopy Chin '04



Momentum resolved rf-spectroscopy measures

spectral function $A_-(\mathbf{k}, \varepsilon_{\mathbf{k}} - \hbar\omega)$ Stewart, Gaebler, Jin '08

removal of an atom to a free one with momentum \mathbf{k}

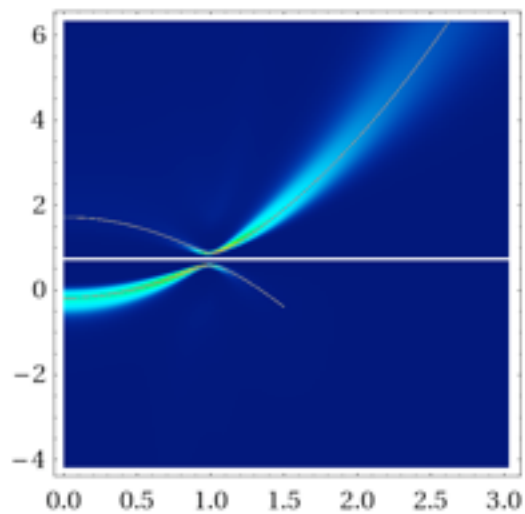
locally resolved rf-spectroscopy Y. Shin,... MIT '08

$$I(\omega) = \hbar \int_{\mathbf{k}} A_-(\mathbf{k}, \varepsilon_{\mathbf{k}} - \hbar\omega) \quad (\text{no final state interactions})$$

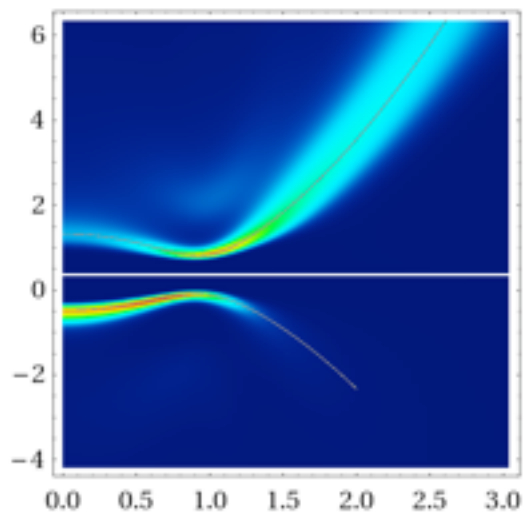
$$A(\mathbf{k}, \varepsilon) \text{ from } \mathcal{G}(\mathbf{k}, \omega_n) = \int d\varepsilon \frac{A(\mathbf{k}, \varepsilon)}{-i\hbar\omega_n + \varepsilon - \mu} \quad (\text{Maxent})$$

BCS-qp's $A_{\text{BCS}}(\mathbf{k}, \varepsilon) = u_{\mathbf{k}}^2 \delta(\varepsilon - E_{\mathbf{k}}^{(+)}) + v_{\mathbf{k}}^2 \delta(\varepsilon - E_{\mathbf{k}}^{(-)})$

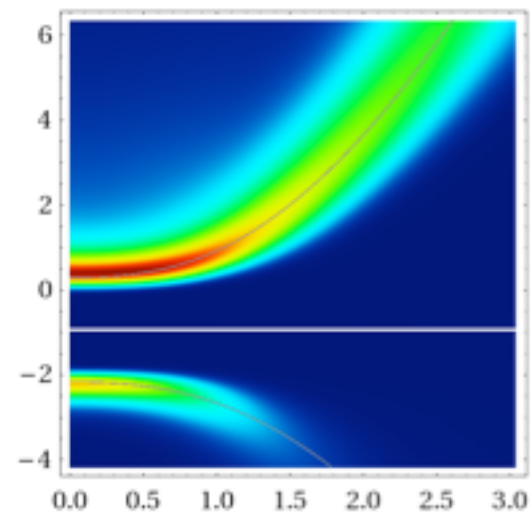
at $E_{\mathbf{k}}^{(\pm)} = \mu \pm \sqrt{(\varepsilon_{\mathbf{k}} - \mu)^2 + \Delta^2}$ have infinite lifetime



$$(k_F a)^{-1} = -1$$

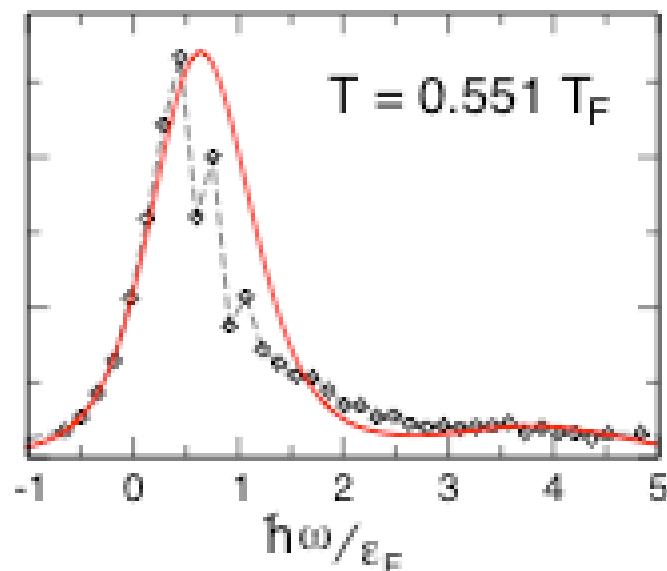
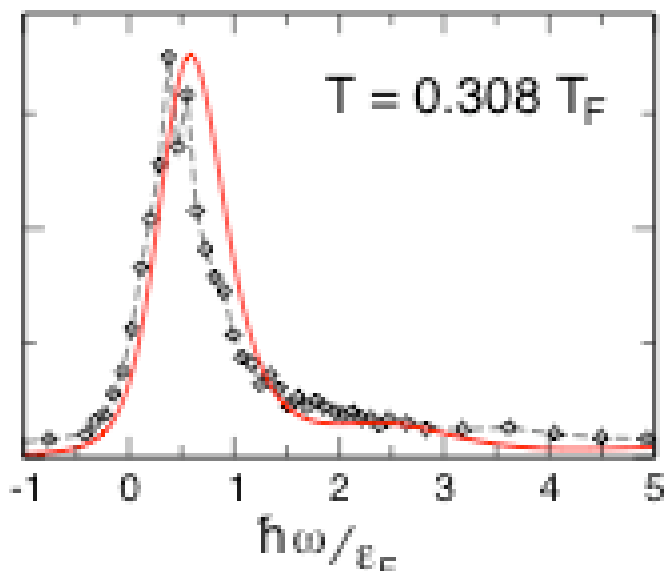
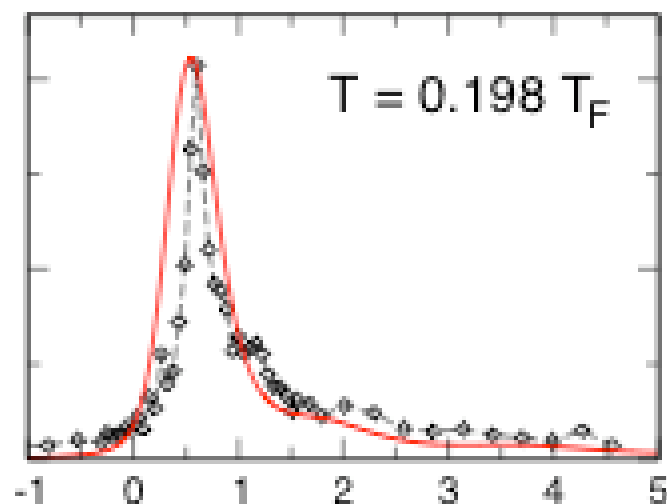
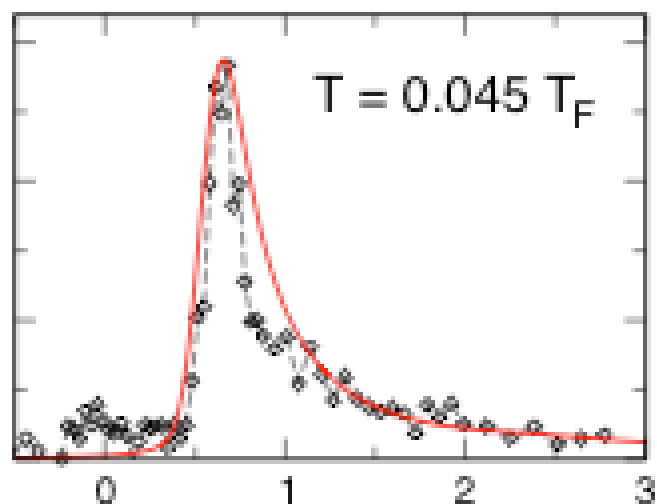


unitarity



$$(k_F a)^{-1} = +1$$

rf-spectra at unitarity (trap center) MIT '08



III) Imbalanced gases

MIT, Rice ... '06

polarization $\sigma = \frac{n_{\uparrow} - n_{\downarrow}}{n_{\uparrow} + n_{\downarrow}}$ induced by $-h(\hat{N}_{\uparrow} - \hat{N}_{\downarrow})$

Clogston-Chandrasekhar field h_c beyond which $\sigma \neq 0$

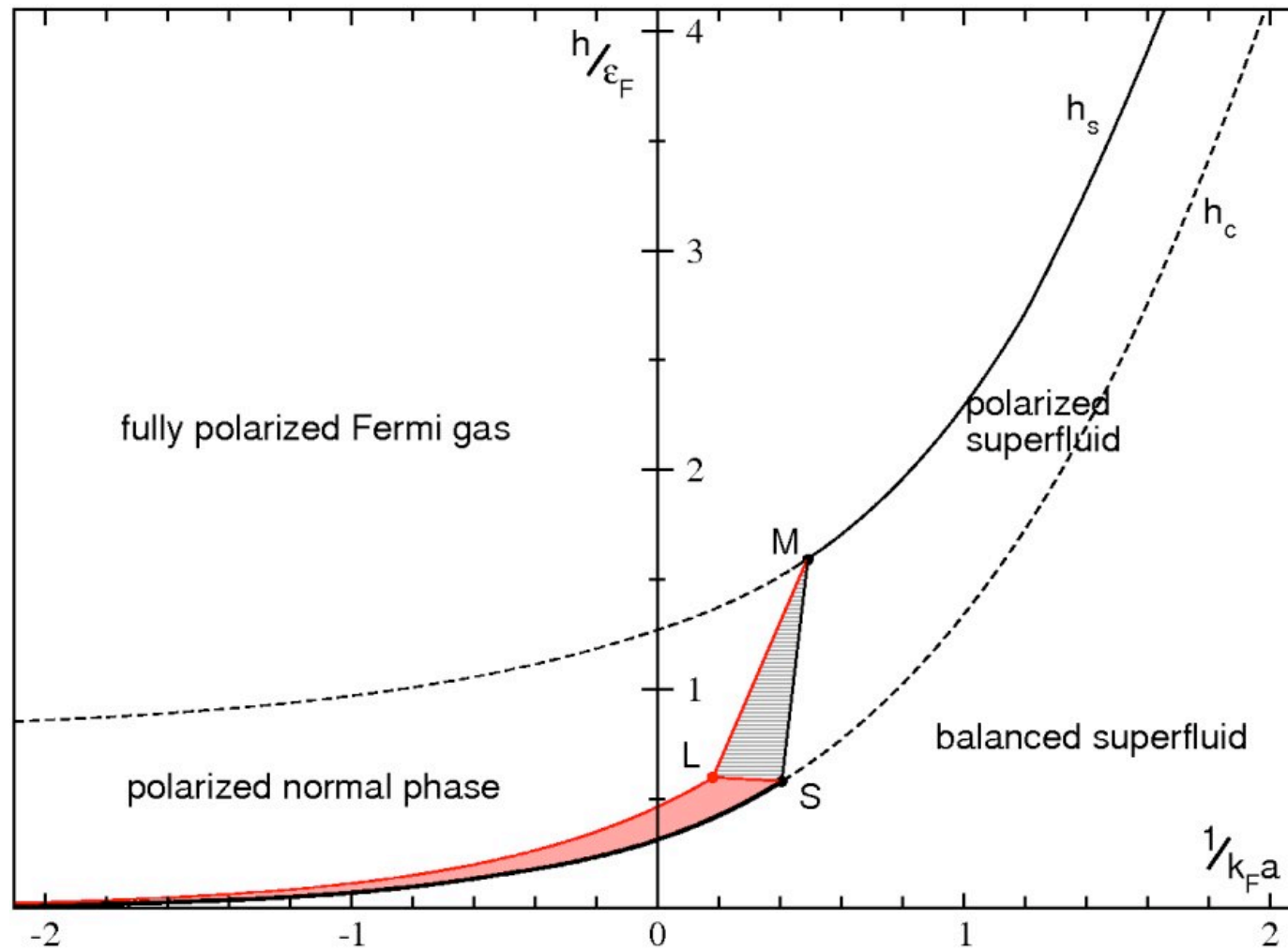
BCS $h_c = \Delta/\sqrt{2}$ ($H_c^{\text{spin}} = 1.84 T \cdot T_c [\text{K}] \gg H_c^{\text{orbit}}$)

BEC $2h_c \rightarrow \varepsilon_b$ energy to break a molecule

saturation field h_s beyond which $\sigma \equiv 1$

BCS $2h_s = \varepsilon_{F\uparrow}$ BEC $2h_s = \varepsilon_b + 2\varepsilon_{F\uparrow} + \dots$

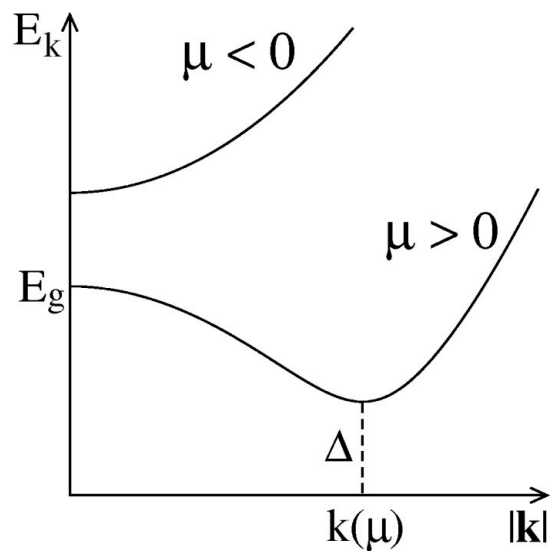
Phase diagram at zero temperature (uniform case)



BCS spectrum has minimum at $\mathbf{k} = 0$ for $\mu < 0$

$$E_k = \sqrt{(\varepsilon_k - \mu)^2 + \Delta^2} \rightarrow E_g + \frac{|\mu|}{E_g} \cdot \varepsilon_k + \dots \quad E_g = \sqrt{\mu^2 + \Delta^2}$$

splitting point S (Son/Stephanov '06) at $1/k_F a|_S \simeq 0.8$



($\mu = 0$ at $1/k_F a = 0.41$!)

BEC side: add \uparrow -Fermions at $\mathbf{k} = 0$

\uparrow -Fermi surface appears contin.

volume $\Omega_{\uparrow} \sim (h - h_c)^{3/2} \sim \sigma$

Strongly imbalanced Fermi gas

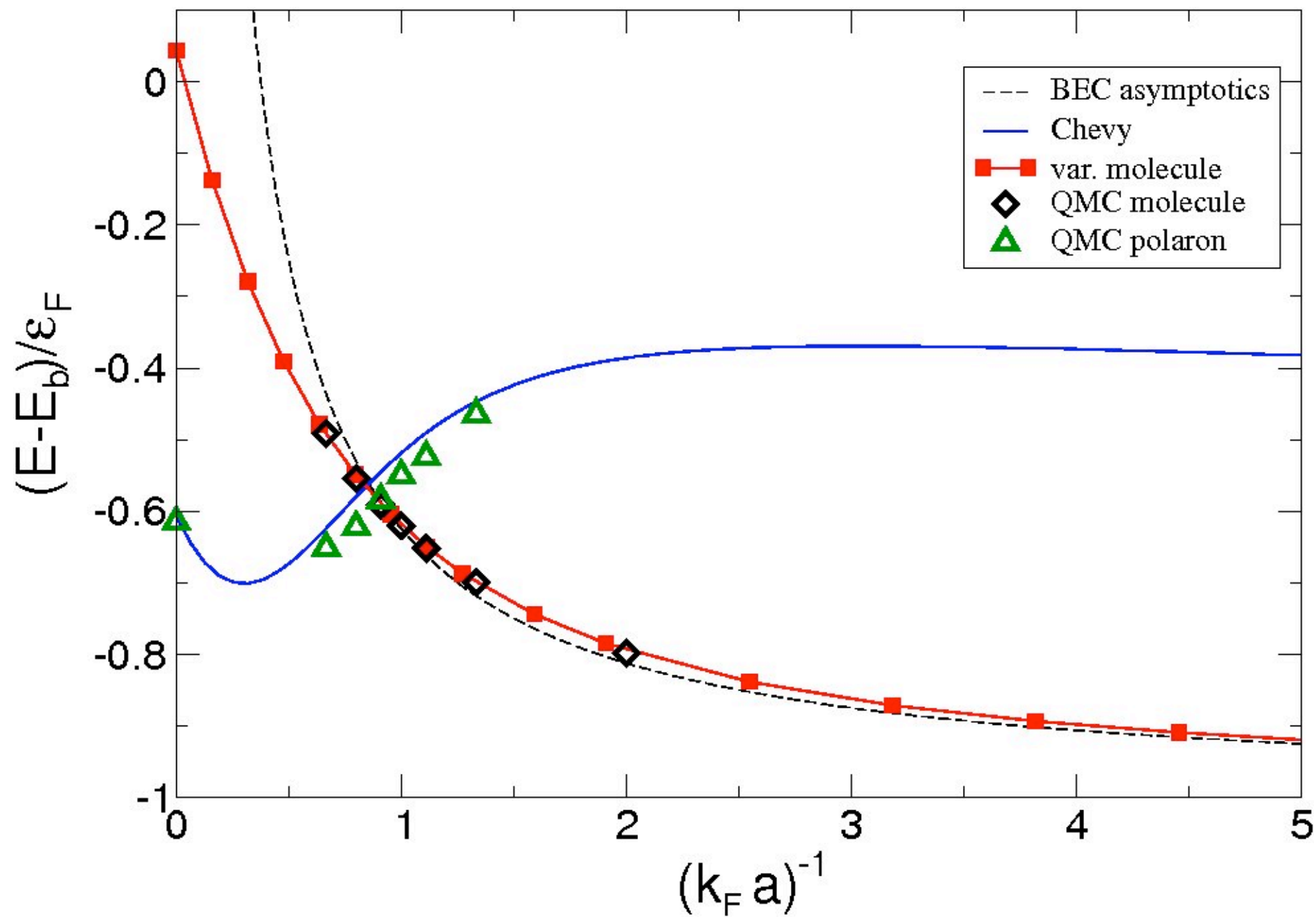
variational wavefunction for a single \downarrow (Chevy '06)

$$|\psi_{\mathbf{p}}\rangle = \phi_0 |\text{FS}\rangle_{\uparrow} |\mathbf{p}\rangle_{\downarrow} + \sum_{\mathbf{k}, \mathbf{q}} \phi_{\mathbf{k}, \mathbf{q}} \hat{c}_{\mathbf{k}\uparrow}^{\dagger} \hat{c}_{\mathbf{q}\uparrow} |\text{FS}\rangle_{\uparrow} |\mathbf{p} + \mathbf{q} - \mathbf{k}\rangle_{\downarrow}$$

μ_{\downarrow} determines **saturation field** $h_s = (\varepsilon_{F,\uparrow} - \mu_{\downarrow})/2$

$|\psi_{\mathbf{p}}\rangle$ is a **Fermi polaron** (Prokof'ev/Svistunov '08)

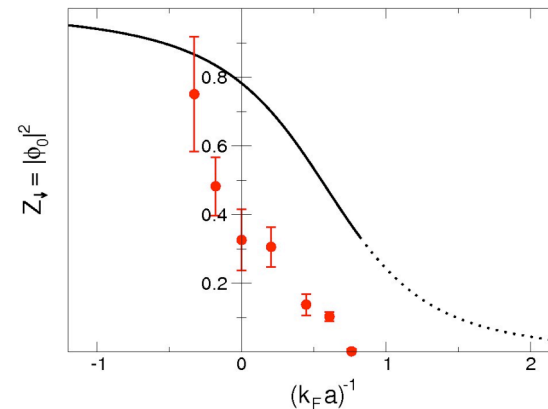
with finite qp-weight $Z_{\downarrow} = |G_{\downarrow}(\mathbf{p} \rightarrow 0, t \rightarrow \infty)| \approx |\phi_0|^2$



rf-spectroscopy for strong imbalance Zwierlein '09

$$I(\omega) \sim Z_{\downarrow} \cdot \delta(\omega + \mu_{\downarrow})$$

crit. coupling $(k_F a)_c^{-1} \simeq 0.96$



Z_{\downarrow} vanishes at a **polaron to molecule transition**

wavefunction for a single $\uparrow\downarrow$ -pair Punk/Zw. '09

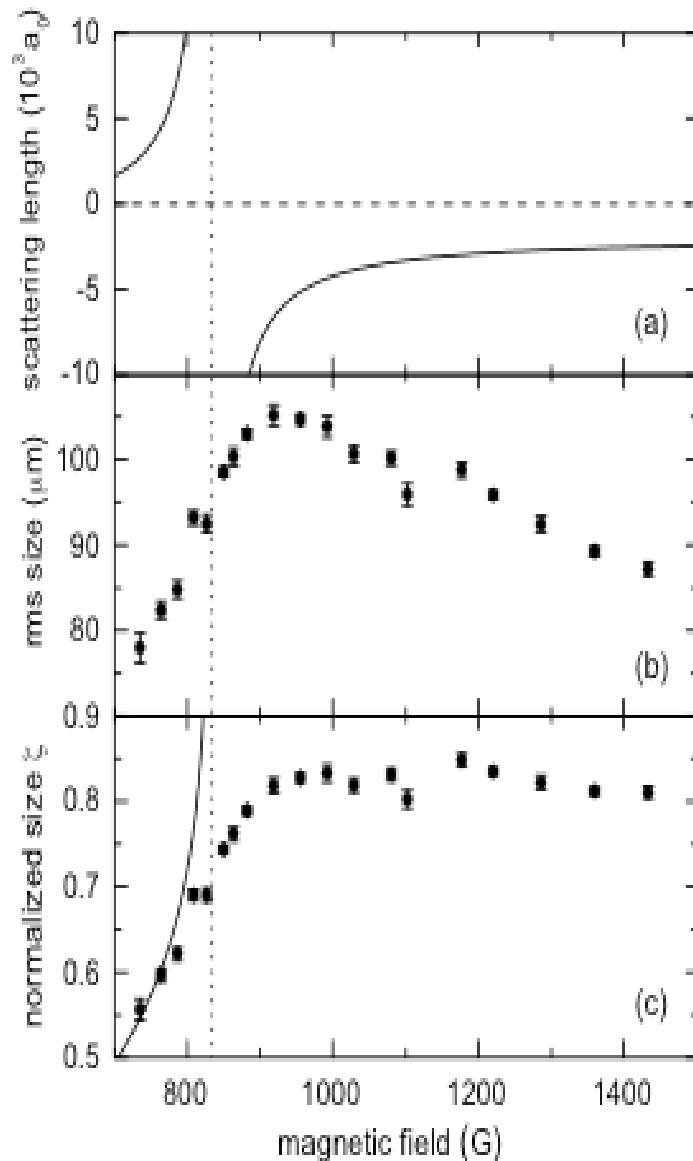
$$\left(\sum \phi_{\mathbf{k}} \hat{c}_{-\mathbf{k}\uparrow}^{\dagger} |\mathbf{k}\rangle_{\downarrow} + \sum \phi_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{c}_{\mathbf{k}\uparrow}^{\dagger} \hat{c}_{\mathbf{k}'\uparrow}^{\dagger} \hat{c}_{\mathbf{q}\uparrow} |\mathbf{q} - \mathbf{k} - \mathbf{k}'\rangle_{\downarrow} \right) |\text{FS}^{(N-1)}\rangle_{\uparrow}$$

has lower ground state energy for $(k_F a)^{-1} \geq 0.84$

Fermi gases near a Feshbach resonance

- realize a high-temperature fermionic superfluid ($T_c/T_F \simeq 0.16$) and a scale- plus conformally invariant many-body problem
- have no well defined fermionic qp's
- are an unsolved problem in the imbalanced case displaying novel phases in a 'simple' system





Bertsch parameter from

cloud size in situ or from

sound velocity $c = v_F \sqrt{\xi/3}$

$$\xi_{\text{exp}} = 0.4 \pm 0.1$$

$$\xi = 0.41 \quad \text{variational MC}$$

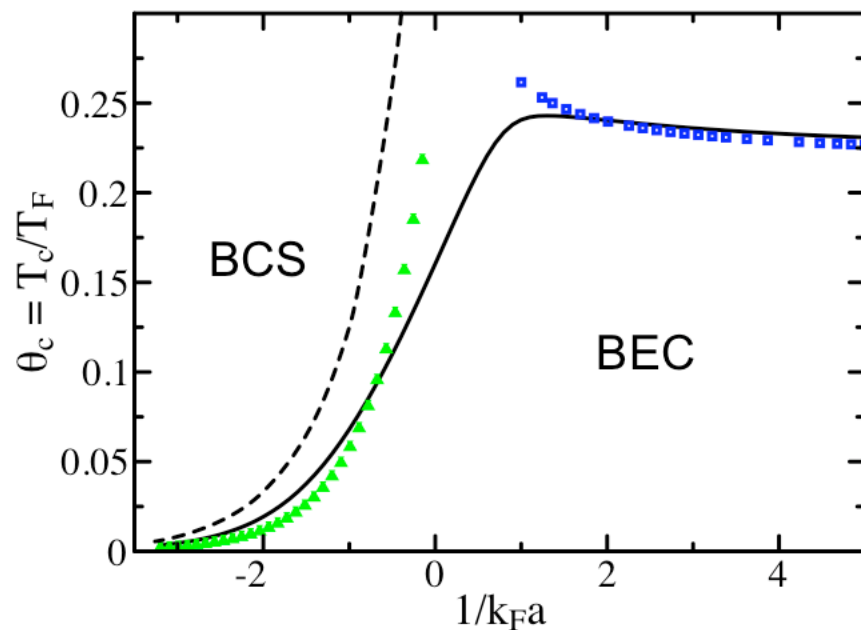
$$\xi = 0.36 \quad \text{Luttinger-Ward}$$

$$\xi = 0.36 \pm 0.002 \quad \text{field theory}$$

field theory $\epsilon = 4 - d$ or $d - 2$ - exp. Nussinov² '04

$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{\epsilon^{5/2}}{16} \ln \epsilon + \dots = 0.36 \pm 0.002 \text{ Nishida/Son '06}$$

$1/N$ -expansion for $2N$ -compon. Nikolic/Sachdev '07



critical temp. $T_c/T_F = 0.16$

Burovski '06 $T_c/T_F = 0.152(7)$

Nishida '07 $T_c/T_F = 0.183$

Nikolic/S. '07 $T_c/T_F = 0.136$

Bulgac '08 $T_c/T_F \lesssim 0.15(1)$

$$I(\omega \rightarrow \infty) = \frac{C}{4\pi^2} \left(\frac{\hbar}{m}\right)^{1/2} \cdot \omega^{-3/2} \quad \text{Schneider '09 det. by}$$

$$\text{Wilson coefficient } C \text{ from } n_\sigma(\mathbf{k}) \rightarrow C/k^4 \quad \text{Tan '05}$$

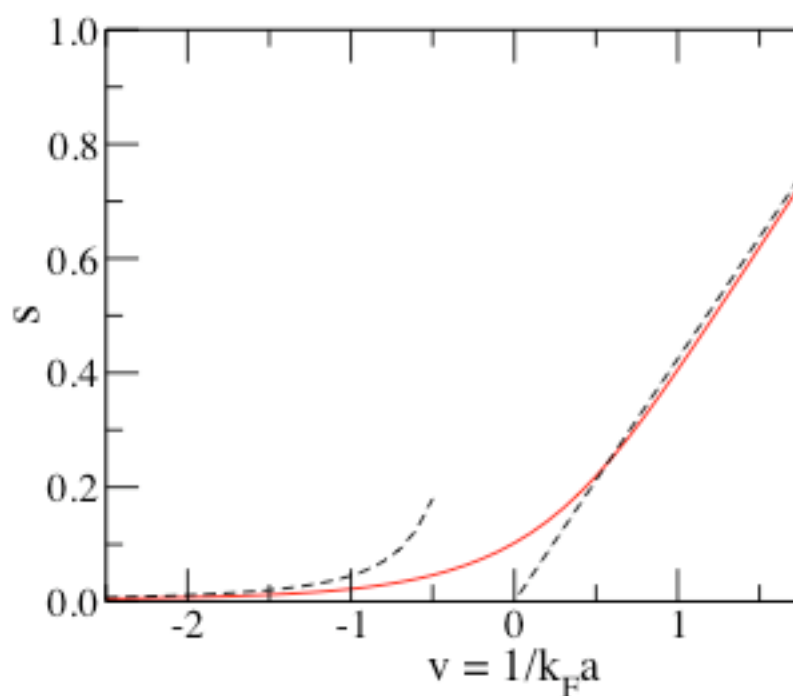
contact density from

$$n_{\uparrow,\downarrow}(r \rightarrow 0) = C \cdot r/4\pi$$

in a sphere of radius r

Braaten/Platter '08

$$(C = k_F^4 \cdot s \text{ balanced gas})$$



Experimental
phase diagram

at unitarity

at finite temp.

Yong-il Shin et.al.

Nature '08

